Chi-Squared Tests 1 MS

Q1.

9	Integrate f(t) to give F(t):	$F(t) = -9/8t^2$	M1		
	Apply limits:	$F(2{\cdot}5) - F(2) = -(9/8)(2{\cdot}5^{-2} - 2^{-2})$	A1		
	Evaluate and multiply by 100:	10·125 **A.G.**	A1	3	
	State hypotheses (A.E.F.):	H_0: f(t) fits data, H_1: doesn't fit	B1		
	Find χ^2 (A1 if at least 3 terms correct):	$\chi^2 = 1{\cdot}5^2/62{\cdot}5 + 4{\cdot}875^2/21{\cdot}875$			
		$+ 5{\cdot}875^2/10{\cdot}125 + 2{\cdot}5^2/5{\cdot}5$	M1 A1		
	Evaluate χ^2:	$= 0{\cdot}036 + 1{\cdot}086 + 3{\cdot}409 + 1{\cdot}136$			
		$= 5{\cdot}67$ [± 0·01]	*A1		
	Compare with consistent tab. value (to 2 dp):	$\chi_{3,\,0.9}^2 = 6{\cdot}251$	*B1√		
	($\chi_{2,\,0.9}^2 = 4{\cdot}605$, $\chi_{1,\,0.9}^2 = 2{\cdot}706$)				
	Consistent conclusion (A1 dep *A1, *B1):	Distribution fits data (A.E.F.)	M1√ A1	7	**[10]**

Q2.

9	Show how value is found (allow M1 if 100 omitted):	$100\ {}^4C_2\ 0{\cdot}4^2 \times 0{\cdot}6^2 = 34{\cdot}56$ **A.G.**	M1 A1	(2)	
	State (at least) null hypothesis:	H_0: B(4, 0·6) fits data (A.E.F.)	B1		
	Combine adjacent cells since exp. value < 5:	O: 14 27 49 10			
		E: 17·92 34·56 34·56 12·96	*M1		
	Calculate value of χ^2 (to 2 dp ; A1 dep *M1):	$\chi^2 = 9{\cdot}22$	M1 A1		
	Compare with consistent tabular value (to 2 dp):	$\chi_{3,\,0.95}^2 = 7{\cdot}815$ (cells combined)			
		$\chi_{4,\,0.95}^2 = 9{\cdot}488$ (not combined)	B1		
	Valid method for reaching conclusion:	Reject H_0 if χ^2 > tabular value	M1		
	Correct conclusion (A.E.F., requires correct values):	9·22 > 7·81[5] so distn. does not fit	*A1		
	State valid deduction (dep *A1; allow $p \neq 0{\cdot}6$):	Prob. of faulty chips ≠ 0·6 (A.E.F.)	B1	(8)	**[10]**

Q3.

8	**(i)**	Find value of k by integrating f(x):	$[\frac{1}{3}kx^3]_0^6 = 1$, $k = 3/6^3 = 1/72$	B1		
		State and evaluate expression for a: **A.G.**	$a = 216\ [\frac{1}{3}kx^3]_2^3 = 3^3 - 2^3 = 19$	B1		
		Find b and c:	$b = 216\ [\frac{1}{3}kx^3]_3^4 = 37$,			
		(**MR**: f(x) as distn. of table: max 3/4)	$c = 216\ [\frac{1}{3}kx^3]_4^5$ *or* 216–155 = 61	M1 A1	4	
	(ii)	State (at least) null hypothesis:	H_0: f(x) fits data (A.E.F.)	B1		
		Combine first 2 cells since exp. value < 5:	O: 4 . . .			
			E: 8 . . .	B1		
		Calculate χ^2 (to 2 dp):	$\chi^2 = 6{\cdot}69$[4]	M1 *A1		
		Compare consistent tabular value (to 2 dp):	$\chi_{4,\,0.9}^2 = 7{\cdot}779$			
		[or if 3 or 0 cells combined:	$\chi_{3,\,0.9}^2 = 6{\cdot}251$, $\chi_{5,\,0.9}^2 = 9{\cdot}236$]	*B1		
		Valid method for reaching conclusion:	Accept H_0 if χ^2 < tabular value	M1		
		Conclusion (A.E.F., dep *A1, *B1):	6·69 < 7·78 so f(x) does fit	A1	7	**[11]**

Chi-Squared Tests 1 MS

Q4.

8	Find mean of sample data for use in Poisson distn.:	$\lambda = \frac{225}{100} = 2{\cdot}25$ B1		
	State (at least) null hypothesis (A.E.F.):	H_0: Poisson distn. fits data B1		
	Find expected values $\frac{100\lambda^r e^{-\lambda}}{r!}$ (to 1 d.p.):	10·540 23·715 26·679 20·009 11·255		
	(ignore incorrect final value here for M1)	5·065 1·899 0·6105 0·2275 M1 A1		
	Combine last four cells so that exp. value ⩾ 5:	O_i: ... 16 14 4 E_i: ... 20·009 11·255 7·802 *M1		
	Calculate value of χ^2 (to 1 d.p.; A1 dep *M1):	$\chi^2 = 1{\cdot}189 + 0{\cdot}582 + 5{\cdot}690 + 0{\cdot}803 + 0{\cdot}6695 + 1{\cdot}853$ $= 10{\cdot}8$ (allow 10·7) M1 A1		
	State or use consistent tabular value (to 1 d.p.):	$\chi^2_{4,\,0.975} = 11{\cdot}14$ (if cells combined) B1 [$\chi^2_{7,\,0.975} = 16{\cdot}01$, $\chi^2_{5,\,0.975} = 12{\cdot}83$]		
	Consistent conclusion (A.E.F., √ on two χ^2 values):	$\chi^2 < 11{\cdot}1$ so Poisson distn. fits B1√	9	**[9]**

Q5.

8	Find mean of sample data [for use in Poisson distn.]:	$\lambda = 220/100 = 2{\cdot}2$ **B1**		
	State (at least) null hypothesis (AEF):	H_0: Poisson distn. fits data *or* $\lambda = 2{\cdot}2$ **B1**		
	Find expected values $100\lambda^r e^{-\lambda}/r!$ (to 1 d.p.):	11·080 24·377 26·814 19·664		
	(ignore incorrect final value here for M1)	10·8151 4·759 2·491 **M1 A1**		
	Combine last two cells so that exp. value ⩾ 5:	O_i: 3 E_i: 7·25 **M1***		
	Calculate value of χ^2 (to 2 d.p.; A1 dep M1*):	$\chi^2 = 0{\cdot}076 + 2{\cdot}879 + 0{\cdot}653 + 1{\cdot}448 + 0{\cdot}441 + 2{\cdot}491$		
	(allow 7·95 if 1 d.p. exp.values used)	$= 7{\cdot}99$ **M1 A1**		
	State or use consistent tabular value (to 3 s.f.):	5 cells: $\chi^2_{3\,,0.95} = 7{\cdot}815$ 6 cells: $\chi^2_{4,\,0.95} = 9{\cdot}488$ (correct) 7 cells: $\chi^2_{5,\,0.95} = 11{\cdot}07$ **B1**		
	State or imply valid method for conclusion e.g.:	Accept H_0 if χ^2 < tabular value **M1**		
	Conclusion (AEF, requires both values correct):	Distn fits *or* $\lambda = 2{\cdot}2$ **A1**	10	**10**
	Not combining cells [so $\chi^2 = 8{\cdot}64$] can earn B1 B1 M1 A1 M0 M1 B1 M1 (max 7)			

Chi-Squared Tests 1 MS

Q6.

9	State (at least) null hypothesis in full: (AEF) Find exp. values using 150 ${}^6C_i\, q^{6-i}\, p^i$ with $p = 0{\cdot}6$, $q = 0{\cdot}4$ (to 3 s.f.): (allow A1 if only one error or if all values correct to 2 s.f.) Combine 0·6144 & 5·5296 since 1st exp. value < 5: Calculate χ^2 (result correct to 3 s.f.): State or use consistent tabular value (to 3 s.f.): [or if no or more cells combined: State or imply valid method for conclusion e.g.: Conclusion (AEF, requires both values correct): [data] (Allow A1 here for e.g. "It is a good fit") **S.C.** 150 ${}^6C_i\, q^i\, p^{6-i}$ can earn B1 M1 B1 M1 B1 M1 (max 6/10)	H_0: Given distribution fits data 0·6144 5·5296 20·736 41·472 46·656 27·9936 6·9984 O_i: 4 . . . E_i: 6·14[4] . . . $\chi^2 = 0{\cdot}748 + 0{\cdot}877 + 2{\cdot}189$ $+ 1{\cdot}606 + 0{\cdot}144 + 3{\cdot}570 = 9{\cdot}13$ 6 cells: $\chi^2_{5,0.95} = 11{\cdot}07$ 7 cells: $\chi^2_{6,0.95} = 12{\cdot}59$ 5 cells: $\chi^2_{4,0.95} = 9{\cdot}488$ 4 cells: $\chi^2_{3,0.95} = 7{\cdot}815$ 3 cells: $\chi^2_{2,0.95} = 5{\cdot}991$] Accept H_0 if χ^2 < tabular value 9·13 [± 0·01] < 11·1 so distn. fits	**B1** **M1 A2** **B1** **M1 A1** **B1**√ **M1** **A1**	[10]

Q7.

11(b)(i)	$x = (1/250)\,\Sigma\, x\, f(x) = 414/250 = 1{\cdot}656$ **AG**	**B1**	Verify given mean
	Total:	**1**	
11(b)(ii)	$p = x/6 = 0{\cdot}276$, $q = 0{\cdot}724$	**M1 A1**	Use 250 ${}^6C_i\, q^{6-i}\, p^i$ and find p and q
	$a = 250\ {}^6C_2\ q^4\, p^2 = 78{\cdot}49 \pm 0{\cdot}01$ (to 2 d.p.)	**A2**	Find either exp. value
	$b = 250\ {}^6C_4\ q^2\, p^4 = 11{\cdot}41 \pm 0{\cdot}01$ (to 2 d.p.)	**A1**	Find other exp. value (deduct single **A1** if either value given to only 1 d.p.)
	Total:	**5**	
11(b)(iii)	H_0: Distribution fits data *or* distribution is binomial (AEF)	**B1**	State (at least) null hypothesis in full Combine values consistent with all exp. values ⩾ 5
	O_i: 48 69 78 32 23 E_i: 36·01 82·36 78·49 39·89 13·26 (± 0·01)	**M1FT A1**	(FT for **M1** but not **A1** on values of a, b)
	$\chi^2 = 3{\cdot}992 + 2{\cdot}167 + 0{\cdot}003 + 1{\cdot}561 + 7{\cdot}154$	**M1**	Find χ^2
	$= 14{\cdot}9$	**A1**	
	No. n of cells: 7 6 5 4 3 $\chi^2_{n-2,0.99}$: 15·09 13·28 11·34 9·210 6·635	**B1FT**	State or use consistent tabular value $\chi^2_{n-2,0.99}$ (to 3 s.f.) [FT on number, n, of cells used to find χ^2]
	Accept H_1 if χ^2 > tabular value (AEF) 14·9 [± 0·1] > 11·34 so distn. doesn't fit [data]	**M1**	State or imply valid method for conclusion Conclusion (requires both values correct)
	or manager's belief not justified (AEF)	**A1**	
	Total:	**8**	