Chi-Squared Tests 1

Goodness of fit tests

Q1.

A sample of 100 observations of the continuous random variable T was obtained and the values are summarised in the following table.

Interval	$1 \leqslant t < 1.5$	$1.5 \leqslant t < 2$	$2 \leqslant t < 2.5$	$2.5 \leqslant t < 3$
Frequency	64	17	16	3

It is required to test the goodness of fit of the distribution with probability density function given by

$$f(t) = \begin{cases} \dfrac{9}{4t^3} & 1 \leqslant t < 3, \\ 0 & \text{otherwise.} \end{cases}$$

The relevant expected values are as follows.

Interval	$1 \leqslant t < 1.5$	$1.5 \leqslant t < 2$	$2 \leqslant t < 2.5$	$2.5 \leqslant t < 3$
Expected frequency	62.5	21.875	10.125	5.5

Show how the expected value 10.125 is obtained. [3]

Carry out the test, at the 10% significance level. [7]

Q2.

It has been found that 60% of the computer chips produced in a factory are faulty. As part of quality control, 100 samples of 4 chips are selected at random, and each chip is tested. The number of faulty chips in each sample is recorded, with the results given in the following table.

Number of faulty chips	0	1	2	3	4
Number of samples	2	12	27	49	10

The expected values for a binomial distribution with parameters $n = 4$ and $p = 0.6$ are given in the following table.

Number of faulty chips	0	1	2	3	4
Expected value	2.56	15.36	34.56	34.56	12.96

Show how the expected value 34.56 corresponding to 2 faulty chips is obtained. [2]

Carry out a goodness of fit test at the 5% significance level, and state what can be deduced from the outcome of the test. [8]

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Q3.

A sample of 216 observations of the continuous random variable X was obtained and the results are summarised in the following table.

Interval	$0 \leqslant x < 1$	$1 \leqslant x < 2$	$2 \leqslant x < 3$	$3 \leqslant x < 4$	$4 \leqslant x < 5$	$5 \leqslant x < 6$
Observed frequency	1	3	15	31	59	107

It is suggested that these results are consistent with a distribution having probability density function f given by

$$f(x) = \begin{cases} kx^2 & 0 \leqslant x < 6, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a positive constant. The relevant expected frequencies are given in the following table.

Interval	$0 \leqslant x < 1$	$1 \leqslant x < 2$	$2 \leqslant x < 3$	$3 \leqslant x < 4$	$4 \leqslant x < 5$	$5 \leqslant x < 6$
Expected frequency	1	7	a	b	c	91

(i) Show that $a = 19$ and find the values of b and c. [4]

(ii) Carry out a goodness of fit test at the 10% significance level. [7]

Q4.

The numbers of a particular type of laptop computer sold by a store on each of 100 consecutive Saturdays are summarised in the following table.

Number sold	0	1	2	3	4	5	6	7	$\geqslant 8$
Number of Saturdays	7	20	39	16	14	2	1	1	0

Fit a Poisson distribution to the data and carry out a goodness of fit test at the 2.5% significance level. [9]

Q5.

The number of goals scored by a certain football team was recorded for each of 100 matches, and the results are summarised in the following table.

Number of goals	0	1	2	3	4	5	6 or more
Frequency	12	16	31	25	13	3	0

Fit a Poisson distribution to the data, and test its goodness of fit at the 5% significance level. [10]

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Q6.

Applicants for a national teacher training course are required to pass a mathematics test. Each year, the applicants are tested in groups of 6 and the number of successful applicants in each group is recorded. The overall proportion of successful applicants has remained constant over the years and is equal to 60% of the applicants. The results from 150 randomly chosen groups are shown in the following table.

Number of successful applicants	0	1	2	3	4	5	6
Number of groups	1	3	25	51	38	30	2

Test, at the 5% significance level, the goodness of fit of the distribution B(6, 0.6) for the number of successful applicants in a group. [10]

Q7.

A shop is supplied with large quantities of plant pots in packs of six. These pots can be damaged easily if they are not packed carefully. The manager of the shop is a statistician and he believes that the number of damaged pots in a pack of six has a binomial distribution. He chooses a random sample of 250 packs and records the numbers of damaged pots per pack. His results are shown in the following table.

Number of damaged pots per pack (x)	0	1	2	3	4	5	6
Frequency	48	69	78	32	22	1	0

(i) Show that the mean number of damaged pots per pack in this sample is 1.656. [1]

The following table shows some of the expected frequencies, correct to 2 decimal places, using an appropriate binomial distribution.

Number of damaged pots per pack (x)	0	1	2	3	4	5	6
Expected frequency	36.01	82.36	a	39.89	b	1.74	0.11

(ii) Find the values of a and b, correct to 2 decimal places [5]

(iii) Use a goodness-of-fit test at the 1% significance level to determine whether the manager's belief is justified. [8]
