

Continuous Random Variables 1

Q1.

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} 0 & x < 1, \\ \frac{1}{2} & 1 \leq x \leq 3, \\ 0 & x > 3. \end{cases}$$

Find the distribution function of X . [2]

The random variable Y is defined by $Y = X^3$. Find

(i) the probability density function of Y , [3]

(ii) the expected value and variance of Y . [3]

Q2.

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{2a} & -a \leq x \leq a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a positive constant. Find the distribution function of X . [3]

Q3.

A continuous random variable X has cumulative distribution function F given by

$$F(x) = \begin{cases} 0 & x < -1, \\ \frac{1}{2}(x^3 + 1) & -1 \leq x \leq 1, \\ 1 & x > 1. \end{cases}$$

Find $P(X \geq \frac{3}{4})$, and state what can be deduced about the upper quartile of X . [3]

Obtain the cumulative distribution function of Y , where $Y = X^2$. [5]

Q4.

The continuous random variable X has distribution function F given by

$$F(x) = \begin{cases} 0 & x < 2, \\ \frac{1}{8}x - \frac{1}{4} & 2 \leq x \leq 10, \\ 1 & x > 10. \end{cases}$$

Find the value of k for which $P(X \geq k) = 0.6$. [3]

The random variable Y is defined by $Y = 2 \ln X$. Find the distribution function of Y . [3]

Find the probability density function of Y and sketch its graph. [4]

Q5.

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{2} & 1 \leq x \leq 3, \\ 0 & \text{otherwise.} \end{cases}$$

The random variable Y is defined by $Y = X^3$. Find the distribution function of Y . [5]

Sketch the graph of the probability density function of Y . [3]

Find the probability that Y lies between its median value and its mean value. [4]

Q6.

8 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{4}(x-1) & 2 \leq x \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X . [3]

The random variable Y is defined by $Y = (X - 1)^3$.

(ii) Find the probability density function of Y . [4]

(iii) Find the median value of Y . [3]

Continuous Random Variables 1

Q7.

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{20} \left(3 - \frac{1}{\sqrt{x}} \right) & 1 \leq x \leq 9, \\ 0 & \text{otherwise.} \end{cases}$$

The random variable Y is defined by $Y = \sqrt{X}$.

(i) Show that the probability density function of Y is given by

$$g(y) = \begin{cases} \frac{1}{10}(3y - 1) & 1 \leq y \leq 3, \\ 0 & \text{otherwise.} \end{cases} \quad [7]$$

(ii) Find the mean value of Y . [2]
