Continuous Random Variables 2 MS

Q1.

6(i)	$F(x) = \int f(x)\,dx = (1/80)(2x^{3/2} - 16x^{1/2})$ [+ c]	M1	Find or state distribution function F(x) for $4 \leqslant x \leqslant 16$ using F(4) = 0 or F(16) = 1 to find c if necessary
	$= (1/80)(2x^{3/2} - 16x^{1/2} + 16)$ *or* $(1/40)(x^{3/2} - 8x^{1/2} + 8)$ *or* $x^{3/2}/40 - x^{1/2}/5 + 1/5$ (AEF)	A1	State F(x) for other values of x
	$F(x) = 0\ (x < 4)$, $F(x) = 1\ (x > 16)$	A1	
		3	
6(ii)	*EITHER:* $G(y)[= P(Y < y) = P(\sqrt{X} < y) = P(X < y^2)]$ $= F(y^2) = (1/40)(y^3 - 8y + 8)$ (AEF)	M1A1	Find or state G(y) for $2 \leqslant y \leqslant 4$ from $Y = \sqrt{X}$ (allow < or $\leqslant$ throughout; **FT** on constant term)
	OR: Use $x = y^2$ to find $f(x) = (1/80)(3y - 8/y)$ *and* $dx/dy = 2y$	(M1A1)	Find f(x) *and* dx/dy for use in $g(y) = f(x) \times \lvert dx/dy \rvert$
	$g(y)\ [= G'(y)] = (1/40)(3y^2 - 8)$ *or* $(3/40)y^2 - 1/5$ [for $2 \leqslant y \leqslant 4$, $g(y) = 0$ otherwise]	A1	Find g(y) in simplified form for $2 \leqslant y \leqslant 4$
		3	

Q2.

7(i)	*EITHER:* $G(y)$ $[= P(Y < y) = P(X^2 < y)$ $= P(X < y^{1/2}) = F(y^{1/2})] = (1/90)(y + y^2)$	M1A1	Find or state G(y) for $0 \leqslant x \leqslant 3$ from $Y = X^2$ (allow < or $\leqslant$ throughout)
	OR: Use $x = y^{1/2}$ to find $f(x) = (1/90)(2x + 4x^3) = (1/90)(2y^{1/2} + 4y^{3/2})$ *and* $dx/dy = 1 / 2y^{1/2}$	(M1A1)	Find f(x) *and* dx/dy for use in $g(y) = f(x) \times \lvert dx/dy \rvert$
	$g(y)\ [= G'(y)] = (1/90)(1 + 2y)$	A1	Find g(y) in simplified form
	for $0 \leqslant y \leqslant 9$ [$g(y) = 0$ otherwise]	A1	State corresponding range of y at any stage
		4	
7(ii)	$E(Y) = (1/90)\int (y + 2y^2)\,dy$	M1	Find mean of Y from $\int y\, g(y)\,dy$
	$= (1/90)[\tfrac{1}{2}y^2 + \tfrac{2}{3}y^3]_0^9 = 117/20$ *or* 5.85	A1	
		2	

Q3.

10(i)	$F(x) = \int f(x)\,dx = (1/30)(-8/x + x^3 - 14x)$ [+ c]	M1	Find or state distribution function F(x) for $2 \leqslant x \leqslant 4$
	$F(x) = (1/30)(-8/x + x^3 - 14x + 24)$	M1	Using F(2) = 0 or F(4) = 1 to find c if necessary. AEF
	$F(x) = 0$ ($x <$ *or* $\leqslant 2$), $F(x) = 1$ ($x >$ *or* $\geqslant 4$)	A1	State F(x) for other values of x
		3	
10(ii)	$G(y) = P(Y < y) = P(X^2 < y)$ $G(y) = P(X < \sqrt{y}) = F(\sqrt{y})$ $G(y) = (1/30)\left(-8/y^{\frac{1}{2}} + y^{\frac{3}{2}} - 14y^{\frac{1}{2}} + 24\right)$	M1 A1	Find or state G(y) for $2 \leqslant x \leqslant 4$ from $Y = X^2$ (allow < or $\leqslant$ throughout)
	Alternative method for question 10(ii)		
	Use $x = y^{\frac{1}{2}}$ to find $f(x) = (1/30)(8/y + 3y - 14), \frac{dx}{dy} = -\frac{1}{2}y^{-\frac{1}{2}}$	(M1 A1)	Find f(x) *and* $\frac{dx}{dy}$ for use in $g(y) = f(x) \times \left\lvert\frac{dy}{dx}\right\rvert$
	$g(y)[= G'(y)] = (1/30)\left(4/y^{\frac{3}{2}} + (3/2)y^{\frac{1}{2}} - 7/y^{\frac{1}{2}}\right)$ for $4 \leqslant y \leqslant 16$ [$g(y) = 0$ otherwise]	A1 A1	Find g(y). AEF State corresponding range of y for G(y) *or* g(y)
		4	
10(iii)	$(1/30)\left(-8/y^{\frac{1}{2}} + y^{\frac{3}{2}} - 14y^{\frac{1}{2}} + 24\right) = 0.8$	M1	Set G(y) = 0·8
	$-8 + y^2 - 14y = 0$, $y = 7 + \sqrt{57}$ *or* 14·5[5] [rejecting $7 - \sqrt{57}$; allow 14·6]	M1 A1	Rearrange to give quadratic in y and solve to find value of y
		3	

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Q4.

3(a)	$\mathrm{E}(X)=\int_0^1 \frac{3}{16}x\left(2-\sqrt{x}\right)\mathrm{d}x+\int_1^9 \frac{\frac{3}{16}x}{\sqrt{x}}\mathrm{d}x$	M1
	$\frac{3}{16}\left[x^2-\frac{2}{5}x^{\frac{5}{2}}\right]+\frac{3}{16}\left[\frac{2}{3}x^{\frac{3}{2}}\right]$	A1
	$\frac{269}{80}$	A1
		3
3(b)	$\mathrm{F}(x)=\begin{cases}\frac{3}{8}\left(x-\frac{1}{3}x^{3/2}\right), & 0\leqslant x<1\\ \frac{3}{8}x^{1/2}-\frac{1}{8}, & 1\leqslant x\leqslant 9\end{cases}$	M1A1
	$\mathrm{G}(y)=\begin{cases}\frac{3}{8}\left(y^2-\frac{1}{3}y^3\right), & 0\leqslant y<1\\ \frac{3}{8}y-\frac{1}{8}, & 1\leqslant y\leqslant 3\end{cases}$	M1A1
	$(\mathrm{G}(y)=0$ for $y\leqslant 0$ and $=1$ for $y\geqslant 3)$ $\mathrm{g}(y)=\begin{cases}\frac{3}{8}\left(2y-y^2\right), & 0\leqslant y<1\\ \frac{3}{8}, & 1\leqslant y\leqslant 3\end{cases}$ $=0$ otherwise	A1
		5

Q5.

3(a)	$\mathrm{F}(x)=\begin{cases}\frac{x^2}{10}, & 0\leqslant x<2\\ \frac{1}{15}\left(10x-x^2\right)-\frac{2}{3}, & 2\leqslant x\leqslant 5\end{cases}$	M1A1
	$=0$ for $x<0$ and $=1$ for $x>5$	A1
		3
3(b)	$\mathrm{F}(m)=½$ so $2m^2-20m+35=0$	M1
	$m=5-\frac{1}{2}\sqrt{30}=2.26$	A1
		2
3(c)	$\mathrm{E}(X^2)=\int_0^2 x^2.\frac{x}{5}\mathrm{d}x+\int_2^5 \frac{2}{15}(5-x).x^2\mathrm{d}x=\left[\frac{x^4}{20}\right]+\left[\frac{2}{15}\left(\frac{5}{3}x^3-\frac{x^4}{4}\right)\right]$	M1
	$=6.5$	A1
		2

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3(d)	F(3) – F(1)	M1
	$= \frac{11}{15} - \frac{1}{10} = \frac{19}{30}$	A1
	Alternative method for 3(d)	
	$\int_1^2 \frac{x}{5}\,dx + \int_2^3 \frac{2}{15}(5-x)\,dx = \left[\frac{x^2}{10}\right] + \left[\frac{2}{15}\left(5x - \frac{x^2}{2}\right)\right]$	M1
	$= \frac{3}{10} + \frac{1}{3} = \frac{19}{30}$	A1
		2

Q6.

3(a)	$\int_0^9 \frac{2}{81} x^{1.5}\,dx$	B1	$f(x) = \frac{2}{81}x$, correct expression as integrand.
	$\frac{2}{81} \times \frac{2}{5} x^{2.5}$	M1	Integrate, FT only on their PDF expression.
	$\frac{12}{5} = 2.4$	A1	
		3	
3(b)	$\int_0^9 \frac{2}{81} x^2\,dx - (their\ a)^2$	M1	
	$\frac{2}{81} \times \frac{9^3}{3} - 2.4^2 = \frac{6}{25}$	A1	
		2	
3(c)	$G(y) = \frac{1}{81} y^6$	M1	
	$g(y) = \frac{2}{27} y^5$	M1	
	Fully correct including 'for $0 \leqslant y \leqslant \sqrt[3]{9}$ and 0 otherwise'	A1	Condone 2.08
		3	