Continuous Random Variables 2

Q1.

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{80}\left(3\sqrt{x} - \frac{8}{\sqrt{x}}\right) & 4 \leqslant x \leqslant 16, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [3]

The random variable Y is defined by $Y = \sqrt{X}$.

(ii) Find the probability density function of Y. [3]

Q2.

The continuous random variable X has distribution function given by

$$F(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{90}(x^2 + x^4) & 0 \leqslant x \leqslant 3, \\ 1 & x > 3. \end{cases}$$

The random variable Y is defined by $Y = X^2$.

(i) Find the probability density function of Y. [4]

(ii) Find the mean value of Y. [2]

Q3.

The random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{30}\left(\frac{8}{x^2} + 3x^2 - 14\right) & 2 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [3]

The random variable Y is defined by $Y = X^2$.

(ii) Find the probability density function of Y. [4]

(iii) Find the value of y such that $P(Y < y) = 0.8$. [3]

Q4.

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{3}{16}\left(2-\sqrt{x}\right) & 0 \leqslant x < 1, \\ \dfrac{3}{16\sqrt{x}} & 1 \leqslant x \leqslant 9, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $E(X)$. [3]

The random variable Y is such that $Y = \sqrt{X}$.

(b) Find the probability density function of Y. [5]

Q5.

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{5}x & 0 \leqslant x < 2, \\ \frac{2}{15}(5-x) & 2 \leqslant x \leqslant 5, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

(b) Find the median value of X. [2]

(c) Find $E(X^2)$. [2]

(d) Find $P(1 \leqslant X \leqslant 3)$. [2]

Q6.

The continuous random variable X has cumulative distribution function F given by

$$F(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{81}x^2 & 0 \leqslant x \leqslant 9, \\ 1 & x > 9. \end{cases}$$

(a) Find $E\left(\sqrt{X}\right)$. [3]

(b) Find $\text{Var}\left(\sqrt{X}\right)$. [2]

(c) The random variable Y is given by $Y^3 = X$. Find the probability density function of Y. [3]
