Continuous Random Variables 3 MS

Q1.

7	(a) (i)	E(X) = 1.5	B1		
		$\frac{2}{9}\int_0^3 (3x^3 - x^4)\mathrm{d}x$	M1		Attempt integ x^2f(x) ignore limits
		$= \frac{2}{9}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3$			
		$= \frac{2}{9}\left[\frac{243}{4} - \frac{243}{5}\right]$ (= 2.7)	M1		Sub correct limits into correct integral
		Var(X) (= $2.7 - 1.5^2$) = 0.45 oe	A1√	[4]	Ft their E(X), but no ft for –ve Var.
	(ii)	0.5	B1	[1]	
	(iii)	$(1 - \frac{13}{27}) \div 2$	M1		or $\frac{2}{9}\int_2^3 (3x - x^2)\ \mathrm{d}x$ oe
		$= \frac{7}{27}$ or 0.259	A1	[2]	As final answer
	(b)	$\frac{1}{2}\times 2\times 2a = \frac{1}{2}$ or $\int_0^2 ax\mathrm{d}x = \frac{1}{2}$	M1		Attempt correct equation in 'a'
		$a = \frac{1}{4}$	A1		
		$\frac{1}{2}\times b\times \frac{1}{4}b = 1$ or $\int_0^b \frac{1}{4}x\mathrm{d}x = 1$ or $b = 2 \times \sqrt{2}$	M1		or $\frac{1}{2}\times b\times ab = 1$ or $\int_0^b ax\mathrm{d}x = 1$ attempt correct equation in (a and) b
		$b = 2\sqrt{2}$	A1√	[4]	Allow $b = \sqrt{8}$ or 2.83 (3 sf) Ft incorrect a, both **M**s needed

Q2.

6	(i)	2 m	B1	[1]	allow without units
	(ii)	$k\int_0^2 x^2(2-x)\mathrm{d}x = 1$	M1		attempt integ f(x) and '= 1'. Ignore limits
		$k\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2$	A1		correct integration and limits
		$k \times \left[\frac{16}{3} - 4\right] = 1$ or $k \times \frac{4}{3} = 1$ oe			
		$k = \frac{3}{4}$ **AG**	A1	[3]	No errors seen
	(iii)	$\frac{3}{4}\int_0^2 x^3(2-x)\mathrm{d}x$	M1		attempt integ xf(x), condone missing k
		$= \frac{3}{4} \times \left[\frac{2x^4}{4} - \frac{x^5}{5}\right]_0^2$	A1		correct integration and limits, condone missing k
		1.2 m oe	A1	[3]	allow without units

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Q3.

5 (i)	$k\int_5^{10}(10t-t^2)\mathrm{d}t=1$	**M1**	Attempt to integrate, ignore limits
	$k\left[5t^2-\frac{t^3}{3}\right]_5^{10}=1$	**A1**	Correct integral and limits
	$k(500-\frac{1000}{3}-(125-\frac{125}{3}))=1$ $k\times\frac{250}{3}=1$ $(k=\frac{3}{250}$ **AG)**	**A1** [3]	No errors seen; No inexact decimals seen
(ii)	$\frac{3}{250}\int_5^{10}(10t^2-t^3)\mathrm{d}t$	**M1**	Attempt to integrate, ignore limits
	$=\frac{3}{250}\left[\frac{10t^3}{3}-\frac{t^4}{4}\right]_5^{10}$	**A1**	Correct integral and limit. Condone missing k
	$=\frac{3}{250}(\frac{10000}{3}-\frac{10000}{4}-(\frac{1250}{3}-\frac{625}{4})$ = 6.875 or 55/8	**A1** [3]	Allow 6.88
(iii)	$\mathrm{P}(T<\mathrm{E}(T)=\frac{3}{250}\left[5t^2-\frac{t^3}{3}\right]_5^{"6.875"}$	**M1***	ft their E(T)
	= 0.5361 "0.5361" – 0.5 P(T between E(T) & median = 0.0361	**DM1*** **A1** [3]	allow 0.036 **Alternative Method** Integrate f(t)limits 5 and m equated to 0.5 M1* Integrate f(t)limits their 6.736 (provided between 5 and 10) and their 6.875DM1 Allow without "minutes"
(iv)	10 (minutes)	**B1** [1]	

Q4.

4(i)	Greater area where x < 7.5 than x > 7.5	**B1**	Allow Graph higher for x < 7.5 than for x > 7.5 or Graph decreasing or equiv expl'n
	Total:	**1**	
4(ii)	$\int_5^{10}\frac{k}{x^2}\mathrm{d}x=1$	**M1**	Attempt Integ f(x) = 1 ignore limits
	$k\left[-\frac{1}{x}\right]_5^{10}=1$ $k\times\frac{1}{10}=1$	**A1**	Correct integration and limits
	$k=10$ AG	**A1**	No errors seen
	Total:	**3**	

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4(iii)	$10\int_5^{10}\frac{1}{x}\,dx$	**M1**	Attempt Integ $xf(x)$ ignore limits
	$= 10\left[\ln x\right]_5^{10}$ $= 10(\ln 10 - \ln 5)$	**M1**	Correct integration and limits
	$= 10\ln 2$ or 6.93 (3 sf)	**A1**	OE
	Total:	**3**	
4(iv)	$10\int_5^{10}1\,dx - "6.93"^2$	**M1**	Attempt (Integ $x^2f(x)$) – $(E(x))^2$. No limits **M0**
	= 1.95 (accept 1.96)	**A1**	*Use of 6.93 gives 1.97* **A0**
	Total:	**2**	

Q5

4(i)	Greater area where $x < 7.5$ than $x > 7.5$	**B1**	Allow Graph higher for $x < 7.5$ than for $x > 7.5$ or Graph decreasing or equiv expl'n
	Total:	**1**	
4(ii)	$\int_5^{10}\frac{k}{x^2}\,dx = 1$	**M1**	Attempt Integ $f(x) = 1$ ignore limits
	$k\left[-\frac{1}{x}\right]_5^{10} = 1$ $k \times \frac{1}{10} = 1$	**A1**	Correct integration and limits
	$k = 10$ AG	**A1**	No errors seen
	Total:	**3**	

4(iii)	$10\int_5^{10}\frac{1}{x}\,dx$	**M1**	Attempt Integ $xf(x)$ ignore limits
	$= 10\left[\ln x\right]_5^{10}$ $= 10(\ln 10 - \ln 5)$	**M1**	Correct integration and limits
	$= 10\ln 2$ or 6.93 (3 sf)	**A1**	OE
	Total:	**3**	
4(iv)	$10\int_5^{10}1\,dx - "6.93"^2$	**M1**	Attempt (Integ $x^2f(x)$) – $(E(x))^2$. No limits **M0**
	= 1.95 (accept 1.96)	**A1**	*Use of 6.93 gives 1.97* **A0**
	Total:	**2**	

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Q6.

4(i)	$k\int_0^a \frac{1}{\sqrt{x}}dx = 1$	**M1**	Attempt int f(x) and = 1 ignore limits
	$(2k[x^{0.5}]_0^a = 1)$ $2ka^{0.5} = 1$ or $a = \frac{1}{4k^2}$	**A1**	OE; a correct eqn in k & a after sub limits
	$k\int_0^a \frac{x}{\sqrt{x}}dx = 3$	**M1**	Attempt int xf(x) and = 3
	e.g. $\frac{2}{3}ka^{1.5} = 3$ or $a^3 = \frac{81}{4k^2}$	**A1**	OE; a correct eqn in k and a after sub limits
	e.g. $a^2 = 81$ or e.g. $k^2 = \frac{81}{4\times 9^3}$	**M1**	Attempt eliminate one letter
	$a = 9$	**A1**	Convincingly obtained
	e.g. $k = \frac{9}{54}$ $k = \frac{1}{6}$ AG	**A1**	
		7	