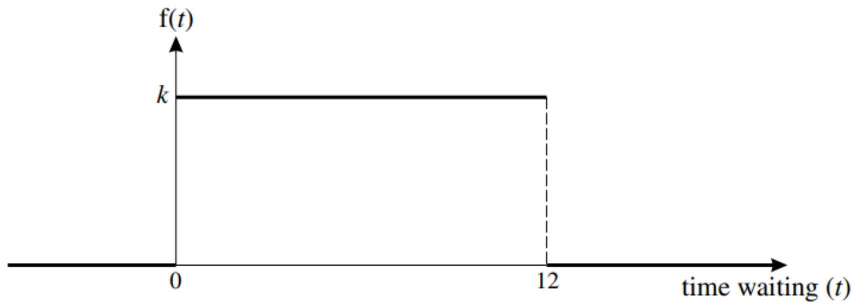
Continuous random variables 1

Q1.

1



Fred arrives at random times on a station platform. The times in minutes he has to wait for the next train are modelled by the continuous random variable for which the probability density function f is shown above.

(i) State the value of k. [1]

(ii) Explain briefly what this graph tells you about the arrival times of trains. [1]

Q2.

A random variable X has probability density function given by

$$f(x) = \begin{cases} k(1-x) & -1 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{2}$. [2]

(ii) Find $P(X > \frac{1}{2})$. [1]

(iii) Find the mean of X. [3]

(iv) Find a such that $P(X < a) = \frac{1}{4}$. [3]

Q3.

5 The time, in minutes, taken by volunteers to complete a task is modelled by the random variable X with probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x^4} & x \geqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that $k = 3$. [2]

(ii) Find $E(X)$ and $Var(X)$. [6]

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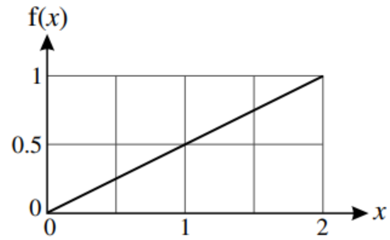
Q4.

4



The diagram shows the graph of the probability density function, f, of a random variable X which takes values between 0 and 2 only.

(i) Find $P(1 < X < 1.5)$. [2]

(ii) Find the median of X. [3]

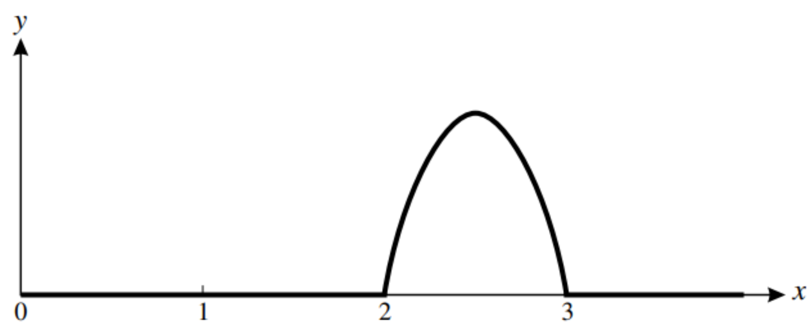
(iii) Find $E(X)$. [2]

Q5.

The distance travelled, in kilometres, by a Grippo brake pad before it needs to be replaced is modelled by $10\,000X$, where X is a random variable having the probability density function

$$f(x) = \begin{cases} -k(x^2 - 5x + 6) & 2 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

The graph of $y = f(x)$ is shown in the diagram.



(i) Show that $k = 6$. [2]

(ii) State the value of $E(X)$ and find $\text{Var}(X)$. [4]

(iii) Sami fits four new Grippo brake pads on his car. Find the probability that at least one of these brake pads will need to be replaced after travelling less than 22 000 km. [3]

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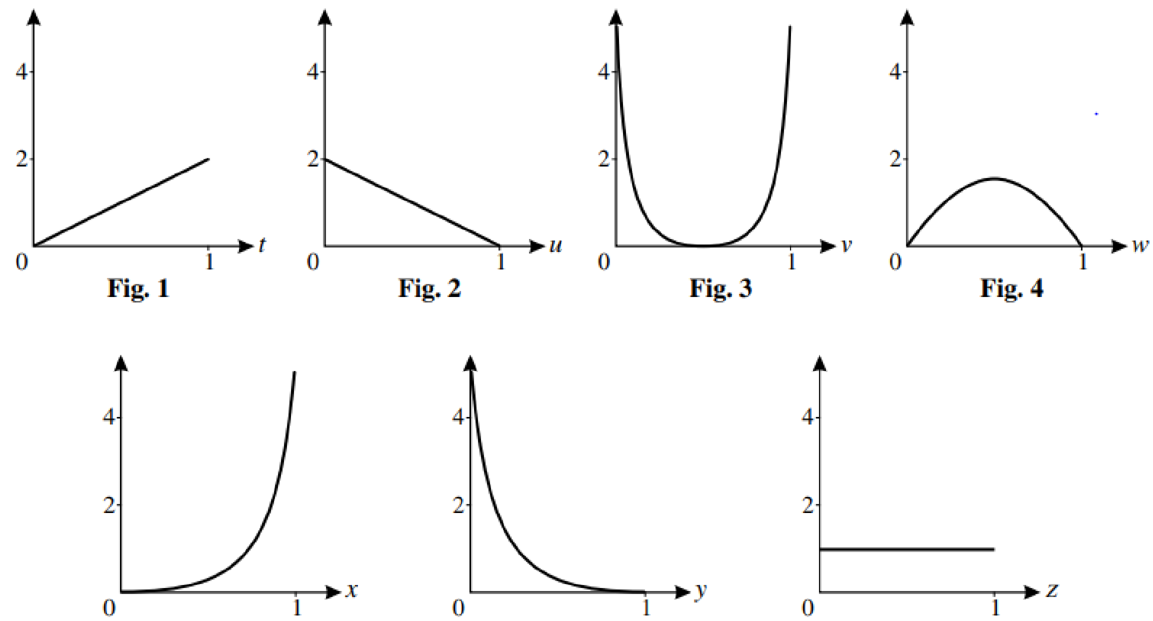
Q6.

7



Each of the random variables T, U, V, W, X, Y and Z takes values between 0 and 1 only. Their probability density functions are shown in Figs 1 to 7 respectively.

(i) **(a)** Which of these variables has the largest median? [1]

(b) Which of these variables has the largest standard deviation? Explain your answer. [2]

(ii) Use Fig. 2 to find $P(U < 0.5)$. [2]

(iii) The probability density function of X is given by

$$f(x) = \begin{cases} ax^n & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where a and n are positive constants.

(a) Show that $a = n + 1$. [3]

(b) Given that $E(X) = \frac{5}{6}$, find a and n. [4]