Continuous Random Variables 2

Q1.

At a certain shop the weekly demand, in kilograms, for flour is modelled by the random variable X with probability density function given by

$$f(x) = \begin{cases} kx^{-\frac{1}{2}} & 4 \leqslant x \leqslant 25, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

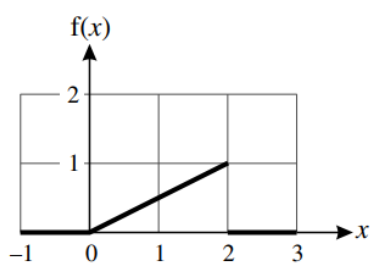
(i) Show that $k = \frac{1}{6}$. [2]

(ii) Calculate the mean weekly demand for flour at the shop. [3]

(iii) At the beginning of one week, the shop has 20 kg of flour in stock. Find the probability that this will not be enough to meet the demand for that week. [2]

(iv) Give a reason why the model may not be realistic. [1]

Q2.



The diagram shows the graph of the probability density function, f, of a random variable X. Find the median of X. [3]

Q3.

Darts are thrown at random at a circular board. The darts hit the board at distances X centimetres from the centre, where X is a random variable with probability density function given by

$$f(x) = \begin{cases} \dfrac{2}{a^2}x & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a positive constant.

(i) Verify that f is a probability density function whatever the value of a. [3]

It is now given that $E(X) = 8$.

(ii) Find the value of a. [3]

(iii) Find the probability that a dart lands more than 6 cm from the centre of the board. [3]

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4.

A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{2}{3}x & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find $E(X)$. [3]

(ii) Find $P(X < E(X))$. [2]

(iii) Hence explain whether the mean of X is less than, equal to or greater than the median of X. [2]

Q5.

The volume, in cm^3, of liquid left in a glass by people when they have finished drinking all they want is modelled by the random variable X with probability density function given by

$$f(x) = \begin{cases} k(x-2)^2 & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{8}$. [2]

(ii) 20% of people leave at least d cm^3 of liquid in a glass. Find d. [3]

(iii) Find $E(X)$. [3]

Q6.

The waiting time, T weeks, for a particular operation at a hospital has probability density function given by

$$f(t) = \begin{cases} \frac{1}{2500}(100t - t^3) & 0 \leqslant t \leqslant 10, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Given that $E(T) = \frac{16}{3}$, find $\text{Var}(T)$. [3]

(ii) 10% of patients have to wait more than n weeks for their operation. Find the value of n, giving your answer correct to the nearest integer. [5]

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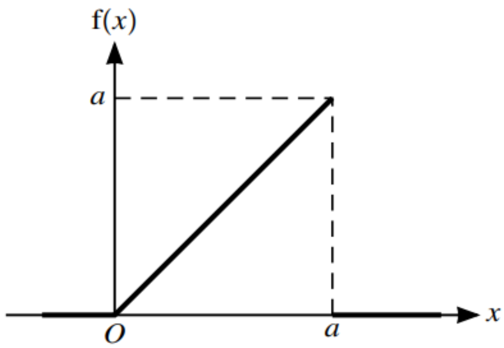
Q7.



The random variable X has probability density function, f, as shown in the diagram, where a is a constant. Find the value of a and hence show that $\mathrm{E}(X) = 0.943$ correct to 3 significant figures. [5]

Q8.

The probability density function of the random variable X is given by

$$\mathrm{f}(x) = \begin{cases} \frac{3}{4}x(c-x) & 0 \leqslant x \leqslant c, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant.

(i) Show that $c = 2$. [3]

(ii) Sketch the graph of $y = \mathrm{f}(x)$ and state the median of X. [3]

(iii) Find $\mathrm{P}(X < 1.5)$. [4]

(iv) Hence write down the value of $\mathrm{P}(0.5 < X < 1)$. [1]
