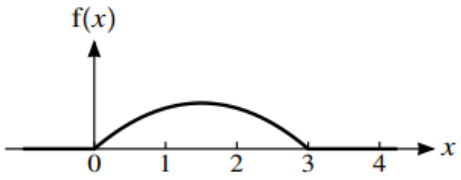
Continuous Random Variables 3

Q1.

(a)



The diagram shows the graph of the probability density function, f, of a random variable X, where

$$f(x) = \begin{cases} \frac{2}{9}(3x - x^2) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(i) State the value of $E(X)$ and find $Var(X)$. [4]

(ii) State the value of $P(1.5 \leqslant X \leqslant 4)$. [1]

(iii) Given that $P(1 \leqslant X \leqslant 2) = \frac{13}{27}$, find $P(X > 2)$. [2]

(b) A random variable, W, has probability density function given by

$$g(w) = \begin{cases} aw & 0 \leqslant w \leqslant b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants. Given that the median of W is 2, find a and b. [4]

Q2.

In each turn of a game, a coin is pushed and slides across a table. The distance, X metres, travelled by the coin has probability density function given by

$$f(x) = \begin{cases} kx^2(2 - x) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) State the greatest possible distance travelled by the coin in one turn. [1]

(ii) Show that $k = \frac{3}{4}$. [3]

(iii) Find the mean distance travelled by the coin in one turn. [3]

(iv) Out of 400 turns, find the expected number of turns in which the distance travelled by the coin is less than 1 metre. [3]

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Q3.

The time, T minutes, taken by people to complete a test has probability density function given by

$$f(t) = \begin{cases} k(10t - t^2) & 5 \leqslant t \leqslant 10, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

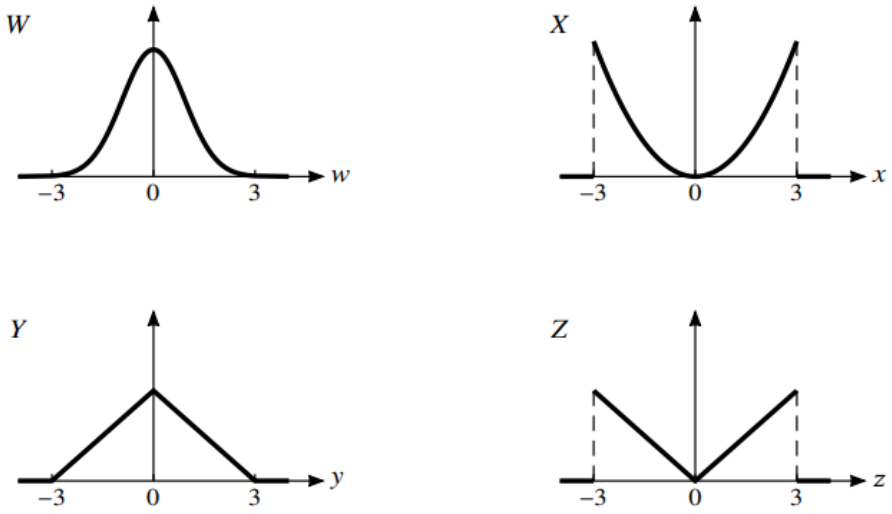
(i) Show that $k = \frac{3}{250}$. [3]

(ii) Find $E(T)$. [3]

(iii) Find the probability that a randomly chosen value of T lies between $E(T)$ and the median of T. [3]

(iv) State the greatest possible length of time taken to complete the test. [1]

Q4.



The diagrams show the probability density functions of four random variables W, X, Y and Z. Each of the four variables takes values between −3 and 3 only, and their standard deviations are σ_W, σ_X, σ_Y and σ_Z respectively.

(i) List σ_W, σ_X, σ_Y and σ_Z in order of size, starting with the largest. [2]

(ii) The probability density function of X is given by

$$f(x) = \begin{cases} \frac{1}{18}x^2 & -3 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $\sigma_X = 2.32$ correct to 3 significant figures. [3]

(b) Calculate $P(X > \sigma_X)$. [3]

(c) Write down the value of $P(X > 2\sigma_X)$. [1]

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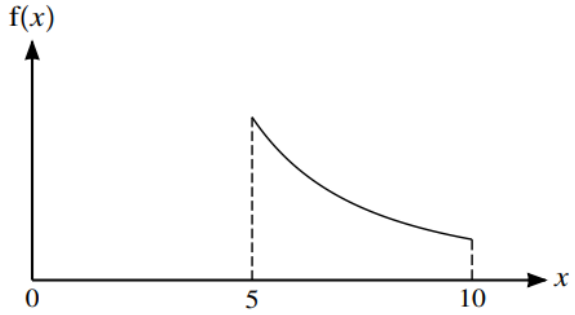
Q5.



The time, X minutes, taken by a large number of runners to complete a certain race has probability density function f given by

$$f(x) = \begin{cases} \frac{k}{x^2} & 5 \leqslant x \leqslant 10, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant, as shown in the diagram.

(i) Without calculation, explain how you can tell that there were more runners whose times were below 7.5 minutes than above 7.5 minutes. [1]

(ii) Show that $k = 10$. [3]

(iii) Find $E(X)$. [3]

(iv) Find $Var(X)$. [2]

Q6.

The random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{k}{\sqrt{x}} & 0 < x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are constants. It is given that $E(X) = 3$.

(i) Find the value of a and show that $k = \frac{1}{6}$. [7]

(ii) Find the median of X. [3]
