Hypothesis Testing 2

Q1.

A cereal manufacturer claims that 25% of cereal packets contain a free gift. Lola suspects that the true proportion is less than 25%. In order to test the manufacturer's claim at the 5% significance level, she checks a random sample of 20 packets.

(i) Find the critical region for the test. [5]

(ii) Hence find the probability of a Type I error. [1]

Lola finds that 2 packets in her sample contain a free gift.

(iii) State, with a reason, the conclusion she should draw. [2]

Q2.

The number of workers, X, absent from a factory on a particular day has the distribution B(80, 0.01).

(i) Explain why it is appropriate to use a Poisson distribution as an approximating distribution for X. [2]

(ii) Use the Poisson distribution to find the probability that the number of workers absent during 12 randomly chosen days is more than 2 and less than 6. [3]

Following a change in working conditions, the management wishes to test whether the mean number of workers absent per day has decreased.

(iii) During 10 randomly chosen days, there were a total of 2 workers absent. Use the Poisson distribution to carry out the test at the 2% significance level. [5]

Q3.

Leila suspects that a particular six-sided die is biased so that the probability, p, that it will show a six is greater than $\frac{1}{6}$. She tests the die by throwing it 5 times. If it shows a six on 3 or more throws she will conclude that it is biased.

(i) State what is meant by a Type I error in this situation and calculate the probability of a Type I error. [3]

(ii) Assuming that the value of p is actually $\frac{2}{3}$, calculate the probability of a Type II error. [3]

Leila now throws the die 80 times and it shows a six on 50 throws.

(iii) Calculate an approximate 96% confidence interval for p. [4]

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Q4.

The heights of a certain variety of plant have been found to be normally distributed with mean 75.2 cm and standard deviation 5.7 cm. A biologist suspects that pollution in a certain region is causing the plants to be shorter than usual. He takes a random sample of n plants of this variety from this region and finds that their mean height is 73.1 cm. He then carries out an appropriate hypothesis test.

(i) He finds that the value of the test statistic z is -1.563, correct to 3 decimal places. Calculate the value of n. State an assumption necessary for your calculation. [4]

(ii) Use this value of the test statistic to carry out the hypothesis test at the 6% significance level. [3]

Q5.

The number of cases of asthma per month at a clinic has a Poisson distribution. In the past the mean has been 5.3 cases per month. A new treatment is introduced. In order to test at the 5% significance level whether the mean has decreased, the number of cases in a randomly chosen month is noted.

(i) Find the critical region for the test and, given that the number of cases is 2, carry out the test. [5]

(ii) Explain the meaning of a Type I error in this context and state the probability of a Type I error. [2]

(iii) At another clinic the mean number of cases of asthma per month has the independent distribution Po(13.1). Assuming that the mean for the first clinic is still 5.3, use a suitable approximating distribution to estimate the probability that the total number of cases in the two clinics in a particular month is more than 20. [5]

Q6.

At the last election, 70% of people in Apoli supported the president. Luigi believes that the same proportion support the president now. Maria believes that the proportion who support the president now is 35%. In order to test who is right, they agree on a hypothesis test, taking Luigi's belief as the null hypothesis. They will ask 6 people from Apoli, chosen at random, and if more than 3 support the president they will accept Luigi's belief.

(i) Calculate the probability of a Type I error. [3]

(ii) If Maria's belief is true, calculate the probability of a Type II error. [3]

(iii) In fact 2 of the 6 people say that they support the president. State which error, Type I or Type II, might be made. Explain your answer. [2]
