Hypothesis Testing 3

Q1.

In the past the weekly profit at a store had mean \$34 600 and standard deviation \$4500. Following a change of ownership, the mean weekly profit for 90 randomly chosen weeks was \$35 400.

(i) Stating a necessary assumption, test at the 5% significance level whether the mean weekly profit has increased. [6]

(ii) State, with a reason, whether it was necessary to use the Central Limit theorem in part **(i)**. [2]

The mean weekly profit for another random sample of 90 weeks is found and the same test is carried out at the 5% significance level.

(iii) State the probability of a Type I error. [1]

(iv) Given that the population mean weekly profit is now \$36 500, calculate the probability of a Type II error. [5]

Q2.

A fair six-sided die has faces numbered 1, 2, 3, 4, 5, 6. The score on one throw is denoted by X.

(i) Write down the value of $\mathrm{E}(X)$ and show that $\mathrm{Var}(X) = \frac{35}{12}$. [2]

Fayez has a six-sided die with faces numbered 1, 2, 3, 4, 5, 6. He suspects that it is biased so that when it is thrown it is more likely to show a low number than a high number. In order to test his suspicion, he plans to throw the die 50 times. If the mean score is less than 3 he will conclude that the die is biased.

(ii) Find the probability of a Type I error. [5]

(iii) With reference to this context, describe circumstances in which Fayez would make a Type II error. [2]

Q3.

Stephan is an athlete who competes in the high jump. In the past, Stephan has succeeded in 90% of jumps at a certain height. He suspects that his standard has recently fallen and he decides to carry out a hypothesis test to find out whether he is right. If he succeeds in fewer than 17 of his next 20 jumps at this height, he will conclude that his standard has fallen.

(i) Find the probability of a Type I error. [4]

(ii) In fact Stephan succeeds in 18 of his next 20 jumps. Which of the errors, Type I or Type II, is possible? Explain your answer. [2]

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Q4.

A researcher is investigating the actual lengths of time that patients spend with the doctor at their appointments. He plans to choose a sample of 12 appointments on a particular day.

(i) Which of the following methods is preferable, and why?

- Choose the first 12 appointments of the day.
- Choose 12 appointments evenly spaced throughout the day. [2]

Appointments are scheduled to last 10 minutes. The actual lengths of time, in minutes, that patients spend with the doctor may be assumed to have a normal distribution with mean μ and standard deviation 3.4. The researcher suspects that the actual time spent is more than 10 minutes on average. To test this suspicion, he recorded the actual times spent for a random sample of 12 appointments and carried out a hypothesis test at the 1% significance level.

(ii) State the probability of making a Type I error and explain what is meant by a Type I error in this context. [2]

(iii) Given that the total length of time spent for the 12 appointments was 147 minutes, carry out the test. [5]

(iv) Give a reason why the Central Limit theorem was not needed in part **(iii)**. [1]

Q5.

A machine is designed to generate random digits between 1 and 5 inclusive. Each digit is supposed to appear with the same probability as the others, but Max claims that the digit 5 is appearing less often than it should. In order to test this claim the manufacturer uses the machine to generate 25 digits and finds that exactly 1 of these digits is a 5.

(i) Carry out a test of Max's claim at the 2.5% significance level. [5]

(ii) Max carried out a similar hypothesis test by generating 1000 digits between 1 and 5 inclusive. The digit 5 appeared 180 times. Without carrying out the test, state the distribution that Max should use, including the values of any parameters. [2]

(iii) State what is meant by a Type II error in this context. [1]

Hypothesis Testing 3

Q6.

A researcher wishes to investigate whether the mean height of a certain type of plant in one region is different from the mean height of this type of plant everywhere else. He takes a large random sample of plants from the region and finds the sample mean. He calculates the value of the test statistic, z, and finds that $z = 1.91$.

(i) Explain briefly why the researcher should use a two-tail test. [1]

(ii) Carry out the test at the 4% significance level. [3]

Q7.

The number of accidents on a certain road has a Poisson distribution with mean 3.1 per 12-week period.

(i) Find the probability that there will be exactly 4 accidents during an 18-week period. [3]

Following the building of a new junction on this road, an officer wishes to determine whether the number of accidents per week has decreased. He chooses 15 weeks at random and notes the number of accidents. If there are fewer than 3 accidents altogether he will conclude that the number of accidents per week has decreased. He assumes that a Poisson distribution still applies.

(ii) Find the probability of a Type I error. [3]

(iii) Given that the mean number of accidents per week is now 0.1, find the probability of a Type II error. [3]

(iv) Given that there were 2 accidents during the 15 weeks, explain why it is impossible for the officer to make a Type II error. [1]
