

Inference with normal and t-distributions 1

Q1.

The speed v at which a javelin is thrown by an athlete is measured in km h^{-1} . The results for 10 randomly chosen throws are summarised by

$$\Sigma v = 1110.8, \quad \Sigma (v - \bar{v})^2 = 333.9,$$

where \bar{v} is the sample mean.

- (i) Stating any necessary assumption, calculate a 99% confidence interval for the mean speed of a throw. [6]

The results for a further 5 randomly chosen throws are now combined with the above results. It is found that the sample variance is smaller than that used in part (i).

- (ii) State, with reasons, whether a 95% confidence interval calculated from the combined 15 results will be wider or less wide than that found in part (i). [2]
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Q2.

A random sample of 10 observations of a normally distributed random variable X gave the following summarised data, where \bar{x} denotes the sample mean.

$$\Sigma x = 70.4 \quad \Sigma (x - \bar{x})^2 = 8.48$$

Test, at the 10% significance level, whether the population mean of X is less than 7.5. [7]

Q3.

A random sample of 9 observations of a normally distributed random variable X gave the following summarised data.

$$\Sigma x = 94.5 \quad \Sigma x^2 = 993.6$$

Test, at the 5% significance level, whether the population mean of X is 10.2. [7]

Calculate a 90% confidence interval for the population mean of X . [3]

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Q4.

The lengths of a random sample of eight fish of a certain species are measured, in cm, as follows.

17.3 15.8 18.2 15.6 16.0 18.8 15.3 15.0

Assuming that lengths are normally distributed,

- (i) test, at the 10% significance level, whether the population mean length of fish of this species is greater than 15.8 cm, [7]
- (ii) calculate a 95% confidence interval for the population mean length of fish of this species. [4]
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Q5.

A random sample of 8 sunflower plants is taken from the large number grown by a gardener, and the heights of the plants are measured. A 95% confidence interval for the population mean, μ metres, is calculated from the sample data as $1.17 < \mu < 2.03$. Given that the height of a sunflower plant is denoted by x metres, find the values of Σx and Σx^2 for this sample of 8 plants. [7]

Q6.

A large number of long jumpers are competing in a national long jump competition. The distances, in metres, jumped by a random sample of 7 competitors are as follows.

6.25 7.01 5.74 6.89 7.24 5.64 6.52

Assuming that distances are normally distributed, test, at the 5% significance level, whether the mean distance jumped by long jumpers in this competition is greater than 6.2 metres. [7]

The distances jumped by another random sample of 8 long jumpers in this competition are recorded. Using the data from this sample of 8 long jumpers, a 95% confidence interval for the population mean, μ metres, is calculated as $5.89 < \mu < 6.75$. Find the unbiased estimates for the population mean and population variance used in this calculation. [5]

Q7.

A random sample of 10 observations of a normal variable X gave the following summarised data, where \bar{x} is the sample mean.

$$\Sigma x = 222.8 \quad \Sigma(x - \bar{x})^2 = 4.12$$

Find a 95% confidence interval for the population mean. [5]
