Inference with normal and t-distributions 2

Q1.

8 (i)	*EITHER:* Estimate *P*'s popln. variance (to 3 d.p.): (allow biased here: 400) Estimate *Q*'s popln. variance (to 3 d.p.): (allow biased here: 314) Find pooled estimate of common variance: *OR:* Find pooled estimate of common variance:	$s_P^2 = (1\,560\,000 - 9600^2/60) / 59$ $[= 406{\cdot}78]$ $s_Q^2 = (1\,052\,500 - 7200^2/50) / 49$ $[= 320{\cdot}41]$ $s^2 = (59\, s_P^2 + 49\, s_Q^2) / 108$ $= 367{\cdot}6$ *or* 368 *or* $9925/27$ $s^2 = (1\,560\,000 - 9600^2/60$ $+ 1\,052\,500 - 7200^2/50) / 108$ $= 367{\cdot}6$ *or* 368 *or* $9925/27$	**M1** **M1** **M1 A1** **(M3 A1)**	[4]
(ii)	Find confidence interval for the difference: Use appropriate tabular value (to 2 d.p.): Evaluate confidence interval (AEF, to 1 d.p.): **SR** Using combined variance $s_P^2/60 + s_Q^2/50 = 13{\cdot}19$: (i) M1, M1 as above; then M0 A0 (max 2/4) (ii) M1 A0 for $9600/60 - 7200/50 \pm z\, s$ A1 for tabular value as above M1 A0 for evaluating interval $16 \pm 7{\cdot}1$ (*or* $7{\cdot}2$) (max 3/5)	$9600/60 - 7200/50$ $[= 160 - 144]$ $\pm z\, s\sqrt{(60^{-1} + 50^{-1})}$ $z_{0.975} = 1{\cdot}96$ *or* $t_{120,\,0.975} = 1{\cdot}98$ $16 \pm 7{\cdot}2$ *or* $[8{\cdot}8, 23{\cdot}2]$ *or* $16 \pm 7{\cdot}3$ *or* $[8{\cdot}7, 23{\cdot}3]$	 **M1 A1** **A1** **M1 A1**	[5]

Q2.

9	$H_0: \mu_X = \mu_Y$, $H_1: \mu_X \neq \mu_Y$ (AEF)	**B1**	State hypotheses (**B0** for $\bar{x}$ …)
	$\bar{x} = 13{\cdot}4/8$ *or* $1{\cdot}67[5]$, $\bar{y} = 2{\cdot}02$ (all to 3 s.f.)	**B1**	Find sample means (values to 3 s.f. throughout)
	$s_X^2 = (24{\cdot}7 - 13{\cdot}4^2/8) / 7$ $= 451/1400$ *or* $0{\cdot}3221$ *or* $0{\cdot}5678^2$ *and* $s_Y^2 = (44{\cdot}6 - 20{\cdot}2^2/10) / 9$ $= 949/2250$ *or* $0{\cdot}4218$ *or* $0{\cdot}6494^2$	**M1**	Estimate or imply popln. variances (allow biased here: $0{\cdot}2819$ *or* $0{\cdot}5309^2$) (allow biased here: $0{\cdot}3796$ *or* $0{\cdot}6161^2$)
	$s^2 = (7\, s_X^2 + 9\, s_Y^2) / 16$ (AEF) *or* $(24{\cdot}7 - 13{\cdot}4^2/8 + 44{\cdot}6 - 20{\cdot}2^2/10) / 16$	**M1 A1**	Estimate (pooled) common variance (note s_X^2 and s_Y^2 not needed explicitly)
	$= 6051/16\,000$ *or* $0{\cdot}3782$ *or* $0{\cdot}6150^2$	**A1**	
	$t_{16,\,0.95} = 1{\cdot}746$	***B1**	State or use correct tabular *t* value
	$[-]\; t = (\bar{y} - \bar{x}) / s\,\sqrt{(1/8 + 1/10)} = 1{\cdot}18$	**M1 A1**	Find value of *t* (or can compare $\bar{y} - \bar{x} = 0{\cdot}345$ with $0{\cdot}509$)
	$t < 1{\cdot}75$ so mean masses are the same (AEF)	**DB1 FT**	Correct conclusion (FT on *t*, dep ***B1**)
	SR: $Z = (\bar{y} - \bar{x}) / \sqrt{(s_X^2/8 + s_Y^2/10)}$ $= 0{\cdot}345 / \sqrt{(0{\cdot}078)} = 1{\cdot}20$	**(B1)**	SR: Implicitly taking s_X^2, s_Y^2 as unequal popln. variances (may also earn first **B1 B1 M1**)
	$Z < 1{\cdot}645$ so mean masses are the same (AEF)	**(B1FT)**	Comparison with $Z_{0.95}$ and conclusion (FT on *Z*) (can earn at most 5/10)
	Total:	**10**	

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Q3.

8	$\bar{x} = 28{\cdot}32$ *and* $\bar{y} = 22{\cdot}2$	B1	Find both sample means
	$s_A^2 = (41\,100 - 1416^2/50) / 49$ *and* $s_B^2 = (20\,140 - 888^2/40) / 39$	M1	Estimate both population variances
	$s_A^2 = 20{\cdot}39$ *and* $s_B^2 = 164/15$ *or* $10{\cdot}93$ (to 3 s.f.)	A1	(allow biased here: 19·98 *and* 10·66)
	EITHER: $s^2 = s_A^2/50 + s_B^2/40 = 0{\cdot}681$ *or* $0{\cdot}825^2$ (19·98/50 + 10·66/40 = 0·666 is M1 A0, max 8/9)	(M1 A1	*EITHER:* Estimate combined variance (if biased values used wrongly here, giving $zs = 1{\cdot}60$, only this A1 is lost)
	$\bar{x} - \bar{y} \pm z\,s$	M1	Find confidence interval for difference
	$z_{0.975} = 1{\cdot}96$	A1	Use appropriate tabular value
	$z\,s = 1{\cdot}62$	A1	Evaluate semi-interval length
	6·12 ± 1·62 *or* [4·50, 7·74]	A1)	State confidence interval (in either form)
	OR: Assume equal [population] variances $s^2 = (49\,s_A^2 + 39\,s_B^2) / 88$ *or* $(41\,100 - 1416^2/50 + 20\,140 - 888^2/40) / 88$	(B1	*OR:* State assumption Find pooled estimate of common variance (**M1 A1** for s_A^2 and s_B^2 may be implied here)
	$= 16{\cdot}2$ *or* $4{\cdot}02^2$	B1	
	$\bar{x} - \bar{y} \pm z\,s\,\sqrt{(1/50 + 1/40)}$	M1	Find confidence interval for difference
	$z_{0.975} = 1{\cdot}96$	A1	Use appropriate tabular z-value (or appropriate t-value from calculator or interpolation)
	($t_{88,\,0.975} = 1{\cdot}9873$ *or* $1{\cdot}99$) 1·67 (1·70)	A1	Evaluate semi-interval length
	6·12 ± 1·67 *or* [4·45, 7·79] (6·12 ± 1·70 *or* [4·42, 7·82])	A1)	Evaluate confidence interval (in either form)
	Total:	**9**	

Q4.

6	$(10 - 4^2/N + 102 - 8^2/2N) / (N + 2N - 2)$ (AEF)	M1 A1	State or find expression for pooled estimate of σ^2 (confusing biased/unbiased estimates may still earn **M1**)
	$112 - 48/N = 10\,(3N - 2)$, $15N^2 - 66N + 24 = 0$	M1 A1	Equate to 10 and rearrange as quadratic
	$N = (66 \pm 54) / 30 = 4$	A1	Solve quadratic for N, rejecting root 0·4
	Total:	**5**	

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Q5.

10(i)	$\bar{x} = 0.91$, $\bar{y} = 1.205$ $s_X^2 = 19.56 / 49$ [= 489/1225 *or* 0.3992] *and* $s_Y^2 = 30.25 / 59$ [= 121/236 *or* 0.5127]	**B1** **M1**	Find both sample means Estimate both population variances (allow biased here: 0.3912 *and* 0.5042)
	EITHER: $s^2 = s_X^2/50 + s_Y^2/60$	**(M1**	Estimate or imply combined variance
	= 0.01653 *or* 0.1286^2 (to 3 s.f. throughout)	**A1**	
	[±] ($\bar{y} - \bar{x}$) ± $z\,s$	**M1**	Find confidence interval for difference $Y - X$ *or* $X - Y$
	$z_{0.95} = 1.645$ [±] 0.295 ± 0.211 (allow 0.212)	**A1**	Use appropriate tabular value Evaluate confidence interval (either form)
	or [±] [0.084, 0.506] (allow [±] [0.083, 0.507])	**A1)**	
	OR: Assume equal [population] variances $s^2 = (49\, s_X^2 + 59\, s_Y^2) / 108$ *or* (19.56 + 30.25) / 108	**(B1**	State assumption Find or imply pooled estimate of common variance (note s_X^2 and s_Y^2 not needed explicitly so first M1 may be implied by result)
	= 4981/10800 *or* 0.461 *or* 0.679^2	**B1**	
	[±] ($\bar{y} - \bar{x}$) ± $z\,s\,\sqrt{(1/50 + 1/60)}$	**M1**	Find confidence interval for difference $Y - X$ *or* $X - Y$
	$z_{0.95} = 1.645$	**A1**	Use appropriate tabular z-value (or t-value from calculator)
	[±] 0.295 ± 0.214 *or* [±] [0.081, 0.509]	**A1)**	Evaluate confidence interval (either form)
		7	

10(ii)	$z = (1.205 - 0.91) / s = 0.295 / s$ = 2.29[4] [or 2.26[9]] (to 3 s.f.)	**M1 A1**	Find value of z (either sign)
	$(z) = 0.989[1]$ [or 0.988[4]]	**A1**	Find $\Phi(z)$
	100 × (1 – 0.989) × 2 = 2.2 [or 2.3] (to 1 d.p.)	**M1 A1**	Find limiting value for α, based on two-tail test (**M0** for basing on one-tail test)
	$\alpha <$ (*or* $\leqslant$) 2.2 [or 2.3]	**A1**	Find set of possible values of α (Treat α instead of α% as misread)
		6	

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Q6.

11O(i)	$ts_A / \sqrt{8} = \frac{1}{2}(16{\cdot}7 - 13{\cdot}5)$ [= 1·6]	M1	Relate s_A to semi-width of confidence interval
	$t_{7,\,0.975} = 2{\cdot}365$ (to 3 s.f.)	A1	State or use correct tabular t value
	[$s_A = \sqrt{8} \times 1{\cdot}6 / 2{\cdot}365 = 1{\cdot}9135$], $s_A{}^2 = 3{\cdot}66$[16]	A1	Hence find unbiased estimate of A's population variance
		3	
11O(ii)	H_0: $\mu_A = \mu_B$, H_1: $\mu_A > \mu_B$ (AEF)	B1	State hypotheses (B0 for $\bar{x}$ …)
	[$\bar{x}_A = 15.1$], $\bar{x}_B = 85.2 / 6 = 14.2$	B1	Find sample mean for B
	$s_B{}^2 = (1221{\cdot}06 - 85{\cdot}2^2/6) / 5$ $= 561/250$ *or* $2{\cdot}244$ *or* $1{\cdot}498^2$ (all to 3 s.f.)	M1	Estimate or imply population variance for B (allow biased here: $1{\cdot}87$ *or* $1{\cdot}367^2$)
	$s^2 = (7\, s_A{}^2 + 5\, s_B{}^2) / 12 = 3{\cdot}0709$ *or* $1{\cdot}752^2$	M1 A1	Estimate (pooled) common variance ($s_B{}^2$ not needed explicitly)
	$t_{12,\,0.95} = 1{\cdot}782$	B1	State or use correct tabular t value
	[–] $t = (\bar{x}_A - \bar{x}_B) / (s\,\sqrt{(1/8 + 1/6)}) = 0{\cdot}951$ $t < 1{\cdot}78$ so [accept H_0]	M1 A1	Find value of t (or can compare $\bar{x}_A - \bar{x}_B = 0{\cdot}9$ with $1{\cdot}69$) Correct conclusion
	mean mass of B not less than mean mass of A (AEF)	B1	
		9	
			SC1: Implicitly taking $s_A{}^2$, $s_B{}^2$ as unequal population variances (may also earn first B1 B1 M1) $z = (\bar{x}_A - \bar{x}_B) / \sqrt{(s_A{}^2/8 + s_B{}^2/6)}$ $= 0{\cdot}9 / \sqrt{(0{\cdot}8317)} = 0{\cdot}987$ $z < 1{\cdot}645$ so
			DepSC1: mean mass of B not less than mean mass of A (AEF) Comparison with $z_{0.95}$ and conclusion (FT on z) (can earn at most 5/9)

Q7.

8	$\bar{x}_A = 32{\cdot}4 / 8 = 4{\cdot}05$	B1	Find sample mean for A
	$s_A{}^2 = (131{\cdot}82 - 32{\cdot}4^2/8) / 7$ $s_A{}^2 = 3/35$ (or $0{\cdot}08571$ or $0{\cdot}2928^2$ both to 3 s.f.)	M1	Estimate or imply popln. variance for A
	H_0: $\mu_A = \mu_B$, H_1: $\mu_A \neq \mu_B$	B1	State hypotheses. AEF
	$s^2 = (7\, s_A{}^2 + 9\, s_B{}^2) / 16 = 0{\cdot}12497$ or $0{\cdot}3535^2$ or $\frac{3999}{32000}$	M1 A1	Estimate (pooled) common variance
	$t_{16,\,0.95} = 1{\cdot}746$	B1*	State or use correct tabular t value
	[–] $t = (x_A - \bar{x}_B) / s\,\sqrt{(1/8 + 1/10)}$	M1	
	$= 0{\cdot}27 / 0{\cdot}1677 = 1{\cdot}61$	A1	Find value of t (or can compare $\bar{x}_A - \bar{x}_B = 0{\cdot}27$ with $0{\cdot}293$)
	$t < 1{\cdot}75$ so [accept H_0] mean masses are the same	DB1	Correct conclusion (FT on t, dep B1*). AEF
		9	