Inference with normal and t-distributions 2

Q1.

The amounts spent on the weekly food shopping by families in the big city P and the small town Q are to be compared. The amounts spent, in dollars, in P and Q are denoted by x and y respectively. For a random sample of 60 families in P and a random sample of 50 families in Q, the amounts are summarised as follows.

$$\Sigma x = 9600 \qquad \Sigma x^2 = 1\,560\,000 \qquad \Sigma y = 7200 \qquad \Sigma y^2 = 1\,052\,500$$

Assuming a common population variance, find

(i) a pooled estimate for the population variance, [4]

(ii) a 95% confidence interval for the difference in the population means in P and Q. [5]

Q2.

Two fish farmers X and Y produce a particular type of fish. Farmer X chooses a random sample of 8 of his fish and records the masses, x kg, as follows.

$$1.2 \quad 1.4 \quad 0.8 \quad 2.1 \quad 1.8 \quad 2.6 \quad 1.5 \quad 2.0$$

Farmer Y chooses a random sample of 10 of his fish and summarises the masses, y kg, as follows.

$$\Sigma y = 20.2 \qquad \Sigma y^2 = 44.6$$

You should assume that both distributions are normal with equal variances. Test at the 10% significance level whether the mean mass of fish produced by farmer X differs from the mean mass of fish produced by farmer Y. [10]

Q3.

The number, x, of beech trees was counted in each of 50 randomly chosen regions of equal size in beech forests in country A. The number, y, of beech trees was counted in each of 40 randomly chosen regions of the same equal size in beech forests in country B. The results are summarised as follows.

$$\Sigma x = 1416 \qquad \Sigma x^2 = 41\,100 \qquad \Sigma y = 888 \qquad \Sigma y^2 = 20\,140$$

Find a 95% confidence interval for the difference between the mean number of beech trees in regions of this size in country A and in country B. [9]

Q4.

The independent variables X and Y have distributions with the same variance σ^2. Random samples of N observations of X and $2N$ observations of Y are taken, and the results are summarised by

$$\Sigma x = 4, \quad \Sigma x^2 = 10, \quad \Sigma y = 8, \quad \Sigma y^2 = 102.$$

These data give a pooled estimate of 10 for σ^2. Find N. [5]

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Q5.

A factory produces bottles of an energy juice. Two different machines are used to fill empty bottles with the juice. The manager chooses a random sample of 50 bottles filled by machine X and a random sample of 60 bottles filled by machine Y. The volumes of juice, x and y respectively, measured in appropriate units, are summarised by

$$\Sigma x = 45.5, \quad \Sigma(x - \bar{x})^2 = 19.56, \quad \Sigma y = 72.3, \quad \Sigma(y - \bar{y})^2 = 30.25,$$

where $\bar{x}$ and $\bar{y}$ are the sample means of the volume of juice in the bottles filled by X and Y respectively.

(i) Find a 90% confidence interval for the difference between the mean volume of juice in bottles filled by machine X and the mean volume of juice in bottles filled by machine Y. [7]

A test at the $\alpha\%$ significance level does not provide evidence that there is any difference in the means of the volume of juice in bottles filled by machine X and the volume of juice in bottles filled by machine Y.

(ii) Find the set of possible values of α. [6]

Q6.

A farmer grows two different types of cherries, Type A and Type B. He assumes that the masses of each type are normally distributed. He chooses a random sample of 8 cherries of Type A. He finds that the sample mean mass is 15.1 g and that a 95% confidence interval for the population mean mass, μ g, is $13.5 \leqslant \mu \leqslant 16.7$.

(i) Find an unbiased estimate for the population variance of the masses of cherries of Type A. [3]

The farmer now chooses a random sample of 6 cherries of Type B and records their masses as follows.

12.2 13.3 16.4 14.0 13.9 15.4

(ii) Test at the 5% significance level whether the mean mass of cherries of Type B is less than the mean mass of cherries of Type A. You should assume that the population variances for the two types of cherry are equal. [9]

Q7.

A random sample of 8 elephants from region A is taken and their weights, x tonnes, are recorded. (1 tonne = 1000 kg.) The results are summarised as follows.

$$\Sigma x = 32.4 \qquad \Sigma x^2 = 131.82$$

A random sample of 10 elephants from region B is taken. Their weights give a sample mean of 3.78 tonnes and an unbiased variance estimate of 0.1555 tonnes^2. The distributions of the weights of elephants in regions A and B are both assumed to be normal with the same population variance. Test at the 10% significance level whether the mean weight of elephants in region A is the same as the mean weight of elephants in region B. [9]
