Inference with normal and t-distributions MS1

Q1.

7 (i)	State suitable assumption (A.E.F.):	Population is Normal	B1		
	Find confidence interval:	$1110{\cdot}8/10 \pm t\sqrt{(333{\cdot}9/90)}$	M1 A1		
		$= 111{\cdot}1 \pm t\sqrt{3{\cdot}71}$	A1		
	State or use correct tabular value of *t*:	$t_{9,0.995} = 3{\cdot}25$	A1		
	Evaluate C.I.:	111 ± 6 *or* [105, 117]	A1	6	
(ii)	Compare *t* , est. variance *s* and *n*:	*t* and *s* smaller, *n* larger	M1		
	Deduce effect on width of C.I. (A.E.F.):	Width is less than in **(i)**	A1	2	**[8]**
	S.R. B1 if valid apart from considering *n*				

Q2.

7	State hypotheses (A.E.F.):	$H_0: \mu = 7{\cdot}5$, $H_1: \mu < 7{\cdot}5$	B1		
	Calculate sample mean:	$\bar{x} = 70{\cdot}4 / 10 = 7{\cdot}04$	M1		
	Estimate population variance:	$s^2 = 8{\cdot}48 / 9 = 211/225$ *or*			
	(allow biased here: $0{\cdot}848$ *or* $0{\cdot}9209^2$)	$0{\cdot}9422$ *or* $0{\cdot}9707^2$	M1		
	Calculate value of *t* (to 3 sf):	$t = (\bar{x} - 7{\cdot}5)/(s/\sqrt{10}) = \pm 1{\cdot}49_{[9]}$	M1 *A1		
	State or use correct tabular *t* value:	$t_{9,\,0.9} = 1{\cdot}38_{[3]}$	*B1		
	(or can compare $\bar{x}$ with $7{\cdot}5 - 0{\cdot}425 = 7{\cdot}07_{[5]}$)				
	correct conclusion (AEF, dep *A1, *B1):	Mean is less than 7·5	B1	7	**[7]**

Q3.

9	Calculate sample mean:	$\bar{x} = 94{\cdot}5 / 9 = 10{\cdot}5$	M1		
	Estimate population variance:	$s^2 = (993{\cdot}6 - 94{\cdot}5^2/9) / 8$			
	(allow biased here: $0{\cdot}15$ *or* $0{\cdot}3873^2$)	$= 0{\cdot}16875$ *or* $0{\cdot}4108^2$	M1		
	State hypotheses (A.E.F.):	$H_0: \mu = 10{\cdot}2$, $H_1: \mu \neq 10{\cdot}2$	B1		
	Calculate value of *t* (to 3 s.f.):	$t = (\bar{x} - 10{\cdot}2)/(s/\sqrt{9}) = 2{\cdot}19$	M1 *A1		
	State or use correct tabular *t* value (to 3 s.f.):	$t_{8,\,0.975} = 2{\cdot}306$	*B1		
	(or can compare $\bar{x}$ with $10{\cdot}2 + 0{\cdot}316 = 10{\cdot}52$)				
	Correct conclusion (AEF, dep *A1, *B1):	Population mean is 10·2	B1	7	
	Find confidence interval (allow *z* in place of *t*) e.g.:	$10{\cdot}5 \pm t\sqrt{\{1{\cdot}35/(8 \times 9)\}}$	M1		
	Use of correct tabular value:	$t_{8,\,0.95} = 1{\cdot}86[0]$	A1		
	Evaluate C.I. correct to 3 s.f.:	$10{\cdot}5 \pm 0{\cdot}255$ *or* [10·2, 10·8]	A1	3	**10**

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Q4.

10 (i)	Calculate sample mean:	$\bar{x} = 132/8 = 16{\cdot}5$ M1		
	Estimate population variance:	$s^2 = (2192{\cdot}06 - 132^2/8)/7$		
	(allow biased here: $1{\cdot}757$ *or* $1{\cdot}326^2$)	$= 703/350$ *or* $2{\cdot}009$ *or* $1{\cdot}417^2$ M1		
	State hypotheses (A.E.F.; B0 for $\bar{x}$):	H_0: $\mu = 15{\cdot}8$, H_1: $\mu > 15{\cdot}8$ B1		
	Calculate value of t (to 3 s.f.):	$t = (\bar{x} - 15{\cdot}8)/(s/\sqrt{8}) = 1{\cdot}4[0]$ M1 A1		
	State or use correct tabular t–value (to 3 s.f.):	$t_{7,\,0.9} = 1{\cdot}41[5]$ B1		
	(or can compare $\bar{x}$ with $15{\cdot}8 + 0{\cdot}709 = 16{\cdot}51$)			
	Consistent conclusion (A.E.F, √ on both t–values):	[Accept H_0:] Popn. mean not greater than $15{\cdot}8$ B1√	7	
(ii)	Find confidence interval (allow z in place of t) e.g.:	$16{\cdot}5 \pm t\sqrt{(2{\cdot}009/8)}$ M1 A1		
	(Use of 15·8 does not lose M1)			
	Use of correct tabular value:	$t_{7,\,0.975} = 2{\cdot}36[5]$ A1		
	Evaluate C.I. correct to 3 s.f.:	$16{\cdot}5 \pm 1{\cdot}18[5]$ *or* $[15{\cdot}3, 17{\cdot}7$ A1	4	**11**

Q5.

7	Find Σx via sample mean $\bar{x}$:	$\Sigma x = 8\bar{x} = 8 \times \frac{1}{2}(1.17 + 2.03)$ $= 8 \times 1.6 = 12.8$ M1 A1		
	Find estimate of population variance s^2:	$t\sqrt{(s^2/8)} = \frac{1}{2}(2.03 - 1.17)$ [= 0.43] M1		
	Use of correct tabular value (1·96 leads to 23·2):	$t_{7,\,0.975} = 2.36[5]$ A1		
	(to 3 d.p.)	$s^2 = 0.2645$ *or* $32/121$ *or* 0.5143^2 A1		
	Find Σx^2 from s^2:	$s^2 = \{\Sigma x^2 - (\Sigma x)^2/8\}/7$		
	(M0 for $s^2 = \{\ldots\}/8$)	$\Sigma x^2 = 7 \times 0.2645 + 12.8^2/8 = 22.3$ M1 A1	7	**7**

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Q6.

8	Calculate sample mean:	$\bar{x} = 45.29 / 7 = 6.47$	M1		
	Estimate population variance:	$s_1^2 = (295.36 - 45.29^2/7) / 6$			
	(allow biased here: 0.3334 *or* 0.5774^2)	$= 0.389$ *or* 0.6236^2	M1		
	State hypotheses: (AEF; B0 for $\bar{x}$)	H_0: $\mu = 6{\cdot}2$, H_1: $\mu > 6.2$	B1		
	Calculate value of t (or $-t$; to 3 s.f.):	$t = (\bar{x} - 6.2)/(s_1/\sqrt{7}) = 1.15$	M1 A1		
	State or use correct tabular t–value (to 3 s.f.):	$t_{6,\,0.95} = 1.94[3]$	B1		
	(or can compare $\bar{x}$ with 6.2 + 0.458 = 6.66)				
	Consistent conclusion (AEF, ft on both t–values):	[Accept H_0]			
		Popn. mean not greater than 6.2	B1√		
	Find estimate of population mean:	½ (5.89 + 6.75) = 6.32	M1 A1	7	
	Find estimate of population variance s_2^2:	$t\sqrt{(s_2^2/8)}$ = ½ (6.75 – 5.89) [= 0.43]	M1		
	SR Allow M1 for $t\sqrt{(s_2^2/7)}$, max 4/5				
	Use of correct tabular value:	$t_{7,\,0.975} = 2.36[5]$	A1		
	(Use of 2.36 may lose final A1)	[$s_2 = 0.5143$ *or* $4\sqrt{2}/11$]			
	(to 3 d.p.)	$s_2^2 = 0.2645$ *or* 32/121	A1	5	**12**

Q7.

5	Find or use sample mean	$\bar{x} = 222{\cdot}8 / 10 = 22{\cdot}28$			
	<u>and</u> estimate population variance:	$s^2 = 4{\cdot}12 / 9$			
	(allow biased here: 0·412 *or* $0{\cdot}642^2$)	$= 0{\cdot}458$ *or* 103/225 *or* $0{\cdot}677^2$	**M1**		
	Find confidence interval (allow z in place of t) e.g.:				
		$22{\cdot}28 \pm t\sqrt{(0{\cdot}458 / 10)}$	**M1 A1**		
	Use of correct tabular value:	$t_{9,\,0.975} = 2{\cdot}26[2]$	**A1**		
	Evaluate C.I. correct to 3 s.f.:	22·3 ± 0·48[4] *or* [21·8, 22·8]	**A1**	5	**5**