Linear Combinations of Random Variables 1 MS

Q1.

4	**(i)** Mr – 5Mrs $\sim N(512 - 5\times89, 62^2 + 25\times7.4^2)$ $\sim N(67, 5213)$	B1 B1	Correct unsimplified mean Correct unsimplified variance
	P(Mr > 5 Mrs) = P(Mr – 5 Mrs > 0) $= P\left(z > \frac{0-67}{\sqrt{5213}}\right)$	M1 M1	Using distribution Mr – 5 Mrs Standardising and using tables
	$= P(z > -0.9280)$ = 0.823	A1 [5]	Correct answer
	(ii) Mr + Mrs $\sim N(601, 62^2 + 7.4^2)$	B1	Correct mean and variance
	E[5/8(Mr + Mrs)] = 376 miles	B1	Correct answer SR Two separate answers 320 and 55.6 B1
	Var[5/8(Mr + Mrs)] = $\frac{25}{64}\times3898.76$ = 1520 sd = 39.0 miles	B1 [3]	Correct answer

Q2.

2	**(i)**	$e^{-\frac{10}{3}} \times \frac{(\frac{10}{3})^4}{4!}$	M1	Allow incorrect λ
		= 0.184 or 0.183	A1 [2]	
	(ii)	$\lambda = 5$	B1	
		$e^{-5}(1 + 5 + \frac{5^2}{2})$	M1	Allow incorrect λ. Allow one end error
		= 0.125 (3 sfs)	A1 [3]	OR Combination method scores B1, identifying all 6 possible combinations M1, multiply each combination and add (must use at least 5 combinations) A1

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Q3.

5	**(i)**	$E(F) = 28 + 1/2 \times 52 = 54$ $Var(F) = 5.6^2 + 1/4 \times 12.4^2$ $= 69.8$	B1 M1 A1 [3]	 $\sqrt{69.8}$ or 8.35: M1A0
	(ii)	H_0: Grinford mean = 54; H_1; Grinford mean < 54 $\dfrac{49-54}{\sqrt{\dfrac{69.8}{10}}}$ $= -1.89(3)$ or $-1,89(2)$ allow + Comp with -1.645 (or 1.893 with 1.645) Evidence that Grinford mean lower	B1ft M1 A1 M1 A1ft [5]	Allow "μ", otherwise undefined mean: B0 ft their 54 Standardising must have $\sqrt{10}$ Comp $P(z < -1.893)$ with 0.05 Allow comparison with 1.96 for consistent 2-tail test Allow "Accept Grinford mean lower" No contradictions OR Alt methods $(x - 54)/(\sqrt{(69.8/10)}) = 1.645$ giving $x = 49.65$ compare with 49 scores M1A1M1A1ft. oe. No mixed methods.

Q4.

4	$E(V) = 46 + 53 + 2 \times 25 = 149$ $Var(V) = 19^2 + 23^2 + 4 \times 10^2$ $= 1290$ $\dfrac{93-149}{\sqrt{'1290'}}$ $= -1.559$ $1 - \Phi('-1.559') = \Phi('1.559')$ $= 0.9405$	B1 M1 A1 M1 A1ft M1 A1 [7]	 or $\sqrt{(19^2 + 23^2 + 4 \times 10^2)}$ or $\sqrt{1290}$ or 35.9 With their mean and their variance. ft their mean and variance providing 3 random variables used, allow +/-. Area consistent with their mean Accept 0.940 or 0.941 or 0.94

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Q5.

5	(i)	$W \sim N(2240, 848)$	B2	B1 each parameter
		$\frac{2200-2240}{\sqrt{848}}$ $(=-1.374)$		
		$\Phi(\text{"}-1.374\text{"}) = 1 - \Phi(\text{"}1.374\text{"})$ $(= 0.0847)$		
		$\frac{2300-2240}{\sqrt{848}}$ $(= 2.060)$		
		$\Phi(\text{"}2.060\text{"})$ $(= 0.9803)$	M1A1	Standardise either value and evaluate correctly
		$\Phi(\text{"}2.060\text{"}) - (1 - \Phi(\text{"}1.374\text{"}))$	M1	Correct combination of Φ's
		$= 0.896$ (3 sfs)	A1 [6]	
	(ii)	$X_1 - X_2 \sim N(0, 392)$	B1	May be implied
		$\frac{20-0}{\sqrt{392}}$ $(= 1.010)$	M1	
		$(\Phi(\text{"}1.010\text{"} = 0.8438)$		
		$P(X > 20) = 1 - \Phi(\text{"}1.010\text{"})$ $(= 0.1562)$	A1	
		$2 \times P(X > 20)$	M1	
		$= 0.312$ (3 sfs)	A1 [5]	

Q6.

1	$(0.7 + 1.0) \times 2$	M1	Attempt combined mean
	$= 3.4$	A1	
	$e^{-3.4}(1 + 3.4 + 3.4^2 \div 2)$	M1	Poisson P(0, 1, 2), any λ (Allow one end error)
	$= 0.34(0)$	A1	
	Alternative Method		
	By Combinations	M2	At least 4 correct $\lambda = 1.4$, $\lambda = 2$
		A1	All 6 correct combinations
		A1 [4]	Correct answer

Q7.

5	(i)	$E(T) = 234$, $Var(T) = 15^2 + 8^2 = 289$	B1	
		$\frac{200-234}{\sqrt{\text{"}289\text{"}}}$ $(=-2.000)$	M1	
		$\Phi(\text{"}-2.000\text{"}) = 1 - \Phi(\text{"}2.000\text{"})$	M1	
		$1 - 0.9772$		
		2.28%	A1 [4]	
	(ii)	Require $P(D > 0)$ where $D = X - 4Y$		
		$E(D)$ $(= 184 - 4 \times 50)$ $= -16$	B1	For -16 or $+16$ or $\pm(184 - 4 \times 50)$
		$Var(D)$ $(= 15^2 + 4^2 \times 8^2)$ $= 1249$	B1	For 1249 or $15^2 + 4^2 \times 8^2$
		$\frac{0-(-16)}{\sqrt{\text{"}1249\text{"}}}$ $(= 0.453)$	M1	
		$1 - \Phi(\text{"}0.453\text{"})$	M1	
		$(= 1 - 0.6747)$		
		$= 0.325$	A1 [5]	

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Q8.

5 (i)	$F + J \sim N(24, 2.8^2 + 2.6^2)$	B1	or N(24, 14.6) for correct mean and variance
	$\frac{30-24}{\sqrt{'14.6'}}$ (= 1.570)	M1	Allow without √ (ignore false cc)
	$P(F + J < 30) = \Phi('1.570')$ 0.942 (3 sfs)	M1 A1 [4]	Correct area consistent with their working
(ii)	$F - 2J \sim N(-11.4, 2.8^2 + 4 \times 2.6^2)$	B1	or N(–11.4, 34.88) for correct mean and variance
	$\frac{0-(-11.4)}{\sqrt{'34.88'}}$ (= 1.930)	M1	Allow without √ (ignore false cc)
	$P(F - 2J) > 0$ $= 1 - \Phi('1.930')$ = 0.0268 (3 sfs)	M1 A1 [4]	Correct area consistent with their working or similar scheme using $2J - F$

Q9.

2 (i)	$E(3X - Y) = 12.1$ $Var(3X - Y) = 9 \times 14 + 15 = (141)$	B1 B1	
	$\frac{20-12.1}{\sqrt{'141'}}$ (= 0.665)	M1	Allow without √ (No Continuity Correction)
	$\Phi('0.665')$ = 0.747 (3 sfs)	M1 A1 5	Correct area consistent with their working

Q10.

3 (i)	Mean = 500 + 3 × 142 = 926 (cents)	B1	
	SD = 3 × 35 = 105 (cents)	M1 A1 [3]	Or 9×35^2 seen Accept √11025
(ii)	Mean = 6 × '926' = 5556 (cents)	B1ft	
	$6 \times '105'^2$ (= 66150)	M1	or SD = √6 × '105'. ft their (i)
	(SD = √66150) = 257 (cents) (3 sf)	A1 [3]	Accept √66150

Q11.

7 (i)	$\lambda = 4.8$	B1	
	$E^{-4.8}(1 + 4.8 + \frac{4.8^2}{2!} + \frac{4.8^3}{3!})$	M1	P(R = 0, 1, 2 or 3), their λ allow one end error
	= 0.294 (3 sfs)	A1 [3]	

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(ii) $e^{-\lambda}\times\frac{\lambda^4}{4!}=\frac{16}{3}e^{-\lambda}\times\frac{\lambda^2}{2!}$ or without $e^{-\lambda}$ $\frac{\lambda^2}{12}=\frac{16}{3}$ or better $(\lambda=8)$ $\lambda=1.6n$ seen or implied $n=$ '8' ÷ 1.6 $=5$	M1 A1 B1 A1 [4]	$\lambda=1.6n$ seen or implied B1 $e^{-1.6n}\times\frac{(1.6n)^4}{4!}=\frac{16}{3}e^{1.6n}\times\frac{(1.6n)^2}{2!}$ M1 $\frac{(1.6n)^2}{12}=\frac{16}{3}$ or better A1 $(1.6n=8)$ $n=5$ A1
(iii) $T\sim N(64, 64)$ $\frac{75.5-64}{\sqrt{64}}$ $(=1.4375)$ $1-\Phi(\text{'}1.4375\text{'})$ $(=1-0.9247)$ $=0.0753$ to 0.0754	B1 M1 M1 A1 [4]	May be implied Allow with wrong or no cc. No sd/var mixes Finding correct area consistent with their working

Q12.

4 **(i)**	$e^{-2}\times2\ (\times)\ e^{-3}\times\frac{3^4}{4!}$	M1		Correct exp'n for P(1) with λ=2 OR P(4) with λ=3
	$e^{-5}\times\frac{5^4}{5!}$	B1		Correct exp'n
	÷			
	$\frac{162}{625}$ or 0.259 (3 sf)	M1		dep M1B1
		A1	4	
(ii)	$(e^{-2}\times\frac{2^r}{r!}=\frac{2}{3}e^{-2}\Rightarrow)$ $3\times2^r=2\times r!$ OR $2^{r-1}=\frac{1}{3}\times r!$	B1		Legitimately shown
	$(\Rightarrow 3\times2^{r-1}=r!)$ $3\times2^3=24$ OR $3!=24$ seen	B1	2	Legitimately shown on either equation

Q13.

2	$(X+Y-Z)\sim N(8,\ldots)$			
	$\mu=8$ (or -8)	B1		seen or implied
	$\text{Var}(X+Y-Z)=2^2+1.5^2+1.8^2$ $(=9.49)$	B1		– award at early stage
	$\frac{0-8}{\sqrt{\text{'}9.49\text{'}}}$ $(=-2.597)$	M1		For standardising (accept sd/var mixes, but variance must be a combination of at least 2 of X, Y, Z)
	$\Phi(\text{'}-2.597\text{'})=1-\Phi(\text{'}2.597\text{'})$	M1		For area consistent with their working
	$=0.0047$	A1	[5]	