Linear Combinations of Random Variables 1

Q1.

The weekly distance in kilometres driven by Mr Parry has a normal distribution with mean 512 and standard deviation 62. Independently, the weekly distance in kilometres driven by Mrs Parry has a normal distribution with mean 89 and standard deviation 7.4.

(i) Find the probability that, in a randomly chosen week, Mr Parry drives more than 5 times as far as Mrs Parry. [5]

(ii) Find the mean and standard deviation of the total of the weekly distances in miles driven by Mr Parry and Mrs Parry. Use the approximation 8 kilometres = 5 miles. [3]

Q2.

People arrive randomly and independently at a supermarket checkout at an average rate of 2 people every 3 minutes.

(i) Find the probability that exactly 4 people arrive in a 5-minute period. [2]

At another checkout in the same supermarket, people arrive randomly and independently at an average rate of 1 person each minute.

(ii) Find the probability that a total of fewer than 3 people arrive at the two checkouts in a 3-minute period. [3]

Q3.

The marks of candidates in Mathematics and English in 2009 were represented by the independent random variables X and Y with distributions $\mathrm{N}(28,\ 5.6^2)$ and $\mathrm{N}(52,\ 12.4^2)$ respectively. Each candidate's marks were combined to give a final mark F, where $F = X + \frac{1}{2}Y$.

(i) Find $\mathrm{E}(F)$ and $\mathrm{Var}(F)$. [3]

(ii) The final marks of a random sample of 10 candidates from Grinford in 2009 had a mean of 49. Test at the 5% significance level whether this result suggests that the mean final mark of all candidates from Grinford in 2009 was lower than elsewhere. [5]

Q4.

The masses, in milligrams, of three minerals found in 1 tonne of a certain kind of rock are modelled by three independent random variables P, Q and R, where $P \sim \mathrm{N}(46,\ 19^2)$, $Q \sim \mathrm{N}(53,\ 23^2)$ and $R \sim \mathrm{N}(25,\ 10^2)$. The total value of the minerals found in 1 tonne of rock is modelled by the random variable V, where $V = P + Q + 2R$. Use the model to find the probability of finding minerals with a value of at least 93 in a randomly chosen tonne of rock. [7]

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Q5.

Cans of drink are packed in boxes, each containing 4 cans. The weights of these cans are normally distributed with mean 510 g and standard deviation 14 g. The weights of the boxes, when empty, are independently normally distributed with mean 200 g and standard deviation 8 g.

(i) Find the probability that the total weight of a full box of cans is between 2200 g and 2300 g. [6]

(ii) Two cans of drink are chosen at random. Find the probability that they differ in weight by more than 20 g. [5]

Q6.

A hotel kitchen has two dish-washing machines. The numbers of breakdowns per year by the two machines have independent Poisson distributions with means 0.7 and 1.0. Find the probability that the total number of breakdowns by the two machines during the next two years will be less than 3. [4]

Q7.

Each drink from a coffee machine contains X cm^3 of coffee and Y cm^3 of milk, where X and Y are independent variables with $X \sim N(184, 15^2)$ and $Y \sim N(50, 8^2)$. If the total volume of the drink is less than 200 cm^3 the customer receives the drink without charge.

(i) Find the percentage of drinks which customers receive without charge. [4]

(ii) Find the probability that, in a randomly chosen drink, the volume of coffee is more than 4 times the volume of milk. [5]

Q8.

Fiona and Jhoti each take one shower per day. The times, in minutes, taken by Fiona and Jhoti to take a shower are represented by the independent variables $F \sim N(12.2, 2.8^2)$ and $J \sim N(11.8, 2.6^2)$ respectively. Find the probability that, on a randomly chosen day,

(i) the total time taken to shower by Fiona and Jhoti is less than 30 minutes, [4]

(ii) Fiona takes at least twice as long as Jhoti to take a shower. [4]

Q9.

The independent random variables X and Y have the distributions $N(6.5, 14)$ and $N(7.4, 15)$ respectively. Find $P(3X - Y < 20)$. [5]

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Q10.

The cost of hiring a bicycle consists of a fixed charge of 500 cents together with a charge of 3 cents per minute. The number of minutes for which people hire a bicycle has mean 142 and standard deviation 35.

(i) Find the mean and standard deviation of the amount people pay when hiring a bicycle. [3]

(ii) 6 people hire bicycles independently. Find the mean and standard deviation of the total amount paid by all 6 people. [3]

Q11.

A random variable X has the distribution $\text{Po}(1.6)$.

(i) The random variable R is the sum of three independent values of X. Find $\text{P}(R < 4)$. [3]

(ii) The random variable S is the sum of n independent values of X. It is given that

$$\text{P}(S = 4) = \tfrac{16}{3} \times \text{P}(S = 2).$$

Find n. [4]

(iii) The random variable T is the sum of 40 independent values of X. Find $\text{P}(T > 75)$. [4]

Q12.

The independent random variables X and Y have the distributions $\text{Po}(2)$ and $\text{Po}(3)$ respectively.

(i) Given that $X + Y = 5$, find the probability that $X = 1$ and $Y = 4$. [4]

(ii) Given that $\text{P}(X = r) = \frac{2}{3}\text{P}(X = 0)$, show that $3 \times 2^{r-1} = r!$ and verify that $r = 4$ satisfies this equation. [2]

Q13.

Each day Samuel travels from A to B and from B to C. He then returns directly from C to A. The times, in minutes, for these three journeys have the independent distributions $\text{N}(20, 2^2)$, $\text{N}(18, 1.5^2)$ and $\text{N}(30, 1.8^2)$, respectively. Find the probability that, on a randomly chosen day, the total time for his two journeys from A to B and B to C is less than the time for his return journey from C to A. [5]
