Linear Combinations of Random Variables 2 MS

Q1.

4 **(i)**	$\lambda = 4.5$	B1		seen
	$1 - e^{-4.5}\left(1+4.5+\frac{4.5^2}{2}\right)$	M1		any λ. Allow one end error
	$= 0.826$ (3 s.f.)	A1	[3]	
(ii)	$e^{-\lambda} = 0.523$	B1		
	$(-\lambda = \ln 0.523)$			
	$\lambda = 0.648$ (3 s.f.)	B1	[2]	
(iii)	$e^{-\mu} \times \frac{\mu^3}{3!} = 24 \times e^{-\mu} \times \mu$	B1		
	$\frac{\mu^2}{6} = 24$	M1		For a simplified expression in μ^2 with $e^{-\mu}$ and μ cancelled and no factorials.
	$\mu = 12$	A1	[3]	

Q2.

1	$N(-35, 60^2 + 4 \times 28^2)$	$N(35, 60^2 + 4 \times 28^2)$	B1 B1		for $\pm(175 - 2 \times 105)$ or ± 35 for $60^2 + 4 \times 28^2$ or 6736
	$\frac{0-(-35)}{\sqrt{'6736'}}$ $(= 0.426)$	$\frac{0-35}{\sqrt{'6736'}}$ $(= -0.426)$	M1		For standardising with their mean and variance. Allow without √ For use of tables and finding area consistent with working
	$1 - \Phi(\text{“}0.426\text{”})$ $= 0.335$ (3 sf)		M1 A1	5	

Q3.

1	$\lambda = (1.2 + 2.3) \div 2$ $= 1.75$	**M1** **A1**		Attempt combined mean, allow 1.2 + 2.3 Correct mean
	$e^{-1.75}\left(\frac{1.75^2}{2} + \frac{1.75^3}{3!}\right)$	**M1**		Allow incorrect mean. Allow end errors (1 and/or 4)
	$= 0.421$ (3 sf)	**A1**	[4]	

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Q4.

7	(i)	E(T) =20.8 Var(T)= $20 \times 0.03^2 + 0.01^2$(= 0.0181) $\frac{20.6-20.8}{\sqrt{"0.0181"}}$ (= –1.487) 1 – Φ(“1.487”) = 0.0684 to 0.686	**B1** **B1** **M1** **M1** **A1** [5]	or $\sqrt{(20 \times 0.03^2 + 0.01^2)} = 0.135$ (3sf) For standardising (σ must come from combination) Area consistent with their working Any answer within range
	(ii)	E(D) = 0 Var(D) = 2×0.0181(= 0.0362) $\frac{0.02-0}{\sqrt{"0.0362)}}$ (= 0.105) Φ(“0.105”) = 0.5418 or 1-Φ(0.015) =0.4582 Φ(“0.105”) – (1 – Φ(“0.105”) (= 0.5418 – 0.4582) = 0.0836/0.0837	**B1**√ **M1** **A1** **M1** **A1** [5]	Both (Seen or implied) Allow without √ Allow to 3sf or 1 – 2(1 – Φ(“0.105”)) (= 1 – 2 × 0.4582)

Q5.

4	(i)	6080 (litres) 106 (litres)	**B1** **B1**	[2]	
	(ii)	E($21Y – 2X$) = 635 Var($21Y – 2X$) = $21^2 \times 12^2 + 2^2 \times 53^2$ (= 74740) $\frac{0-635}{\sqrt{'74740'}}$ (= –2.323) 1–Φ(‘–2.323’) = Φ(‘2.323’) = 0.99(0) (3 sf)	**B1** **B1** **M1** **M1** **A1**	[5]	correct expression or result or sd = 273 seen no sd/var mixes Area consistent with their working No errors seen

Q6.

3	N(178, …) Var = $3.2^2 + 4.1^2 + 3.8^2$ or 41.49 $\frac{175-'178'}{\sqrt{'41.49'\div\sqrt{15}}}$ (= ‘–1.804’) Φ(‘–1.804’) = 1 – Φ (‘1.804’) = 0.0356 (3 sf)	**B1** **B1** **M1** **M1** **A1**	[5]	stated or implied or sd = 6.44 stated or implied need √15 but allow var / sd mix for “41.49” allow cc for method marks independent M1 for area / prob consistent with working

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Q7.

7 (i)	$1 - e^{-1}(1 + 1)$ (= 0.26424) $1 - e^{-1.5}(1 + 1.5 + \frac{1.5^2}{2!})$ (= 0.19115)	**B1** **B1**		B1 for either λ correct. B1 for either correct expression with correct λ
	'0.26424' × '0.19115'	**M1**		product of their values for ⩽ 2 and ⩽ 3 from Poisson, need correct form " 1 - .. " , but allow incorrect λ values and end errors
	= 0.0505 (3 sf)	**A1**	[4]	accept 0.0504

(ii)	λ = 30 N(30, 30)	**B1** **B1**√		seen or implied, need N(λ,λ)
	$\frac{35.5-30}{\sqrt{30}}$ (= 1.004)	**M1**		allow with wrong or no cc or no √
	Φ ('1.004')	**M1**		consistent with their working
	= 0.842 (3 sf)	**A1**	[5]	

Q8.

5(i)	$W \sim N(6210, 171.88)$	**B2**	seen or implied. **B1** each parameter
	$\frac{6200 - "6210"}{\sqrt{"171.88"}}$ (= – 0.763)	**M1**	Standardising with their values. No sd / var mix
	1 – Φ("0.763")	**M1**	For area consistent with their mean
	= 0.223 (3 sfs)	**A1**	
	Total:	**5**	

5(ii)	E(C – 2B) = −50	**M1**	"6210"–2(3130) (or E(2B–C)=50
	Var(C – 2B) = "171.88" + $2^2 \times 12.1^2$ (= 757.52)	**M1**	
	$\frac{0 - (-50)}{\sqrt{"757.52"}}$ (= 1.817)	**M1**	Standardising with their values
	Φ("1.817")	**M1**	For area consistent with their mean
	= 0.965 (3 sfs)	**A1**	
	Total:	**5**	

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Q9.

6(i)	Mean = $3.2 \times 90 = 288$	**B1**	
	Variance = $0.4^2 \times 90^2$	**M1**	
	$= 1296$	**A1**	
		3	
6(ii)	Mean = '288' + $4.3 \times 95 = 696.5$	**B1 FT**	
	Variance = '1296' + $0.6^2 \times 95^2 = 4545$	**B1 FT**	FT their **(i)**
	$\frac{670-696.5}{\sqrt{4545}}$ (= -0.393)	**M1**	FT Var provided both given Vars used standardising (ignore cc) no sd / Var mix
	$1 - \phi('-0.393') = \phi('0.393')$	**M1**	correct area consistent with their working (i.e. their mean)
	= 0.653 (3 sf)	**A1**	
		5	

Q10.

5(i)	$T_1 + T_2 \sim N(5, 0.4^2 + 0.5^2)$	**B1**	or N(5, 0.41)
	$\frac{6-5}{\sqrt{'0.41'}}$ (= 1.562)	**M1**	Allow cc
	$\Phi('1.562')$	**M1**	Correct area consistent with their working
	= 0.941	**A1**	
		4	
5(ii)	$\text{Var}(T_2 - 1.2T_1) = 0.5^2 + 1.2^2 \times 0.4^2$ (= 0.4804)	**B1**	Or similar using $1.2T_1 - T_2$
	$T_2 - 1.2T_1 \sim N(0.16, 0.4804)$	**B1 ft**	Only ft attempt at combination. no ft for neg var.
	$\frac{0-'0.16'}{\sqrt{'0.4804'}}$ (= -0.231)	**M1**	Standardise with their mean and variance. Allow cc
	$P(T_2 - 1.2T_1) > 0$		
	$= \Phi('0.231')$	**M1**	Correct area consistent with their working
	= 0.591 (3 sfs)	**A1**	
		5	

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Q11.

5(i)	$1-e^{-1.8}(1+1.8)$	**M1**	Accept any λ. Accept 1–P(0,1,2)
	$= 0.537$ (3 sf)	**A1**	
		2	
5(ii)	$\lambda = 2.2$	**B1**	
	$e^{-2.2}(1+2.2+\frac{2.2^2}{2!}+\frac{2.2^3}{3!}+\frac{2.2^4}{4!})$	**M1**	Attempt expr'n for $P(X \leqslant 4)$,allow one end error, allow any λ
	$= 0.928$ (3 sf) or 0.927	**A1**	
		3	

5(iii)	$1 - e^{-1.8t} \geqslant 0.99$ or $1 - e^{-\lambda} \geqslant 0.99$	**M1**	Condone = signs/incorrect inequality signs
	$e^{-1.8t} \leqslant 0.01$ or $e^{-\lambda} \leqslant 0.01$ $-1.8t \leqslant \ln 0.01$	**M1**	Valid attempt take logs (must have single term on each side)
	$t \geqslant 2.56$ She must watch for at least 2.56 (hours)	**A1**	or 2 hours, 34 mins or better. No errors seen
		3	

Q12.

5(i)	mean = 3250 var. = 61	**B1**	Or mean = 325 var. $= \frac{6.1}{10}$
	$\frac{3240-3250}{\sqrt{61}}(=-1.280)$	**M1**	Standardise with their values (no mixed methods)
	$\phi('-1.280') = 1 - \phi('1.280')$	**M1**	Area consistent with their figures
	0.100	**A1**	Allow 0.1
		4	
5(ii)	$E(D) = 325 - 2 \times 167 = -9$	**B1**	Accept ± 9
	$Var(D) = 6.1 + 2^2 \times 5.6 (= 28.5)$	**B1**	
	$\frac{0-(-9)}{\sqrt{28.5}}(=1.686)$	**M1**	Standardising with *their* values. Must have a combination attempt on denominator and $\sqrt{}$
	$1 - \phi('1.686')$	**M1**	Area consistent with their figures
	0.0459	**A1**	
		5	