Linear Combinations of Random Variables 2

Q1.

(i) The random variable W has the distribution Po(1.5). Find the probability that the sum of 3 independent values of W is greater than 2. [3]

(ii) The random variable X has the distribution Po(λ). Given that $P(X = 0) = 0.523$, find the value of λ correct to 3 significant figures. [2]

(iii) The random variable Y has the distribution Po(μ), where $\mu \neq 0$. Given that

$$P(Y = 3) = 24 \times P(Y = 1),$$

find μ. [3]

Q2.

The masses, in grams, of tomatoes of type A and type B have the distributions $N(125, 30^2)$ and $N(130, 32^2)$ respectively.

(i) Find the probability that the total mass of 4 randomly chosen tomatoes of type A and 6 randomly chosen tomatoes of type B is less than 1.5 kg. [5]

(ii) Find the probability that a randomly chosen tomato of type A has a mass that is at least 90% of the mass of a randomly chosen tomato of type B. [5]

Q3.

Failures of two computers occur at random and independently. On average the first computer fails 1.2 times per year and the second computer fails 2.3 times per year. Find the probability that the total number of failures by the two computers in a 6-month period is more than 1 and less than 4. [4]

Q4.

Bags of sugar are packed in boxes, each box containing 20 bags. The masses of the boxes, when empty, are normally distributed with mean 0.4 kg and standard deviation 0.01 kg. The masses of the bags are normally distributed with mean 1.02 kg and standard deviation 0.03 kg.

(i) Find the probability that the total mass of a full box of 20 bags is less than 20.6 kg. [5]

(ii) Two full boxes are chosen at random. Find the probability that they differ in mass by less than 0.02 kg. [5]

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Q5.

Each week a farmer sells X litres of milk and Y kg of cheese, where X and Y have the independent distributions $N(1520, 53^2)$ and $N(175, 12^2)$ respectively.

(i) Find the mean and standard deviation of the total amount of milk that the farmer sells in 4 randomly chosen weeks. [2]

During a year when milk prices are low, the farmer makes a loss of 2 cents per litre on milk and makes a profit of 21 cents per kg on cheese, so the farmer's overall weekly profit is $(21Y - 2X)$ cents.

(ii) Find the probability that, in a randomly chosen week, the farmer's overall profit is positive. [5]

Q6.

A men's triathlon consists of three parts: swimming, cycling and running. Competitors' times, in minutes, for the three parts can be modelled by three independent normal variables with means 34.0, 87.1 and 56.9, and standard deviations 3.2, 4.1 and 3.8, respectively. For each competitor, the total of his three times is called the race time. Find the probability that the mean race time of a random sample of 15 competitors is less than 175 minutes. [5]

Q7.

Men arrive at a clinic independently and at random, at a constant mean rate of 0.2 per minute. Women arrive at the same clinic independently and at random, at a constant mean rate of 0.3 per minute.

(i) Find the probability that at least 2 men and at least 3 women arrive at the clinic during a 5-minute period. [4]

(ii) Find the probability that fewer than 36 people arrive at the clinic during a 1-hour period. [5]

Q8.

Large packets of sugar are packed in cartons, each containing 12 packets. The weights of these packets are normally distributed with mean 505 g and standard deviation 3.2 g. The weights of the cartons, when empty, are independently normally distributed with mean 150 g and standard deviation 7 g.

(i) Find the probability that the total weight of a full carton is less than 6200 g. [5]

Small packets of sugar are packed in boxes. The total weight of a full box has a normal distribution with mean 3130 g and standard deviation 12.1 g.

(ii) Find the probability that the weight of a randomly chosen full carton is less than double the weight of a randomly chosen full box. [5]

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Q9.

The numbers of barrels of oil, in millions, extracted per day in two oil fields A and B are modelled by the independent random variables X and Y respectively, where $X \sim N(3.2, 0.4^2)$ and $Y \sim N(4.3, 0.6^2)$. The income generated by the oil from the two fields is \$90 per barrel for A and \$95 per barrel for B.

(i) Find the mean and variance of the daily income, in millions of dollars, generated by field A. [3]

(ii) Find the probability that the total income produced by the two fields in a day is at least \$670 million. [5]

Q10.

The times, in months, taken by a builder to build two types of house, P and Q, are represented by the independent variables $T_1 \sim N(2.2, 0.4^2)$ and $T_2 \sim N(2.8, 0.5^2)$ respectively.

(i) Find the probability that the total time taken to build one house of each type is less than 6 months. [4]

(ii) Find the probability that the time taken to build a type Q house is more than 1.2 times the time taken to build a type P house. [5]

Q11.

The number of eagles seen per hour in a certain location has the distribution Po(1.8). The number of vultures seen per hour in the same location has the independent distribution Po(2.6).

(i) Find the probability that, in a randomly chosen hour, at least 2 eagles are seen. [2]

(ii) Find the probability that, in a randomly chosen half-hour period, the total number of eagles and vultures seen is less than 5. [3]

Alex wants to be at least 99% certain of seeing at least 1 eagle.

(iii) Find the minimum time for which she should watch for eagles. [3]

Q12.

The masses, in grams, of large boxes of chocolates and small boxes of chocolates have the distributions N(325, 6.1) and N(167, 5.6) respectively.

(i) Find the probability that the total mass of 10 randomly chosen large boxes of chocolates is less than 3240 g. [4]

(ii) Find the probability that the mass of a randomly chosen large box of chocolates is more than twice the mass of a randomly chosen small box of chocolates. [5]
