Matrices and Transformations 1

Q1. The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 5 & k \\ -3 & -4 \end{pmatrix}.$$

(a) Find the value of k for which $\mathbf{A}$ is singular. [2]

It is now given that $k = 6$ so that $\mathbf{A} = \begin{pmatrix} 5 & 6 \\ -3 & -4 \end{pmatrix}$.

(b) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by $\mathbf{A}$. [6]

(c) The triangle DEF in the x-y plane is transformed by $\mathbf{A}$ onto triangle PQR.

(i) Given that the area of triangle DEF is $10\,\text{cm}^2$, find the area of triangle PQR. [2]

(ii) Find the matrix which transforms triangle PQR onto triangle DEF. [2]

Q2. Let $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$.

(a) The transformation in the x-y plane represented by $\mathbf{A}^{-1}$ transforms a triangle of area $30\,\text{cm}^2$ into a triangle of area $d\,\text{cm}^2$.

Find the value of d. [3]

(b) Prove by mathematical induction that, for all positive integers n,

$$\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 2^n - 1 & 1 \end{pmatrix}.$$ [5]

(c) The line $y = 2x$ is invariant under the transformation in the x-y plane represented by $\mathbf{A}^n\mathbf{B}$, where $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 33 & 0 \end{pmatrix}$.

Find the value of n. [5]

Q3. The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} k & 0 & 2 \\ 0 & -1 & -1 \\ 1 & 1 & -k \end{pmatrix},$$

where k is a real constant.

(a) Show that **A** is non-singular. [3]

The matrices **B** and **C** are given by

$$\mathbf{B} = \begin{pmatrix} 0 & -3 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} -3 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

It is given that $\mathbf{CAB} = \begin{pmatrix} -2 & -\frac{3}{2} \\ -1 & -\frac{3}{2} \end{pmatrix}$.

(b) Find the value of k. [3]

(c) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by **CAB**. [5]
