Matrices and Transformations 1 - MS

Q1.

(a)	$-20+3k=0$	1	**M1**
	$k=\frac{20}{3}$	1	**A1**
		2	
(b)	Consider $\begin{pmatrix}5 & 6\\-3 & -4\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}5x+6y\\-3x-4y\end{pmatrix}\ \left(=\begin{pmatrix}X\\Y\end{pmatrix}\right)$	1	**M1**
	Line through O, so $y=mx$	1	**M1**
	Invariant line, so $Y=mX$, $\Rightarrow -3x-4mx=m(5x+6mx)$	2	**M1A1**
	$6m^2+9m+3=0$ $m=-1,-\frac{1}{2}$	1	**A1**
	Invariant lines $y=-x$, $2y+x=0$	1	**A1**
		6	
(c)(i)	det **A** $=-20+18=-2$	1	**M1**
	Area $=2\times10=20\,\text{cm}^2$	1	**A1**
		2	
(c)(ii)	$\mathbf{A}^{-1}=\frac{1}{-2}\begin{pmatrix}-4 & -6\\3 & 5\end{pmatrix}=\begin{pmatrix}2 & 3\\-\frac{3}{2} & -\frac{5}{2}\end{pmatrix}$	**2**	**M1A1**

Q2.

(a)	$\det\mathbf{A}^{-1}=(\det\mathbf{A})^{-1}=\frac{1}{2}$	**M1 A1**
	$d=15$	**A1**
		3
(b)	$\mathbf{A}=\begin{pmatrix}2 & 0\\1 & 1\end{pmatrix}=\begin{pmatrix}2^1 & 0\\2^1-1 & 1\end{pmatrix}$ so true when $n=1$.	**B1**
	Assume that it is true for $n=k$, so $\mathbf{A}^k=\begin{pmatrix}2^k & 0\\2^k-1 & 1\end{pmatrix}$.	**B1**
	Then $\mathbf{A}^{k+1}=\begin{pmatrix}2^k & 0\\2^k-1 & 1\end{pmatrix}\begin{pmatrix}2 & 0\\1 & 1\end{pmatrix}=\begin{pmatrix}2^{k+1} & 0\\2(2^k-1)+1 & 1\end{pmatrix}=\begin{pmatrix}2^{k+1} & 0\\2^{k+1}-1 & 1\end{pmatrix}$	**M1A1**
	So, it is also true for $n=k+1$. Hence, by induction, true for all positive integers.	**A1**
		5
(c)	$\mathbf{A}^n\mathbf{B}=\begin{pmatrix}2^n & 0\\2^n-1 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\33 & 0\end{pmatrix}=\begin{pmatrix}2^n & 0\\2^n+32 & 0\end{pmatrix}$	**M1A1**
	$\begin{pmatrix}2^n & 0\\2^n+32 & 0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}2^n x\\(2^n+32)x\end{pmatrix}$	**B1**
	$(2^n+32)x=2^{n+1}x\Rightarrow 2^n=32\Rightarrow n=5$	**M1 A1**
		5

Q3.

(a)	$k\begin{vmatrix}-1 & -1\\ 1 & -k\end{vmatrix}+2\begin{vmatrix}0 & -1\\ 1 & 1\end{vmatrix}=0 \Rightarrow k^2+k+2=0$	M1 A1
	$1-4(2)=-7<0 \Rightarrow$ Non-singular	A1
		3
(b)	$\begin{pmatrix}-3 & -1 & 1\\ 1 & 1 & 2\end{pmatrix}\begin{pmatrix}k & 0 & 2\\ 0 & -1 & -1\\ 1 & 1 & -k\end{pmatrix}\begin{pmatrix}0 & -3\\ -1 & 3\\ 0 & 0\end{pmatrix}=\begin{pmatrix}-3 & -1 & 1\\ 1 & 1 & 2\end{pmatrix}\begin{pmatrix}0 & -3k\\ 1 & -3\\ -1 & 0\end{pmatrix}$	M1 A1
	$=\begin{pmatrix}-2 & 9k+3\\ -1 & -3k-3\end{pmatrix}=\begin{pmatrix}-2 & -\frac{3}{2}\\ -1 & -\frac{3}{2}\end{pmatrix}\Rightarrow k=-\frac{1}{2}$	A1
		3
(c)	$\begin{pmatrix}2 & \frac{3}{2}\\ 1 & \frac{3}{2}\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}2x+\frac{3}{2}y\\ x+\frac{3}{2}y\end{pmatrix}$	B1
	$x+\frac{3}{2}mx=m\left(2x+\frac{3}{2}mx\right)$	M1 A1
	$1+\frac{3}{2}m=2m+\frac{3}{2}m^2 \Rightarrow 3m^2+m-2=0$	A1
	$y=-x$ and $3y-2x=0$	A1
		5