Matrices and Transformations 2 - MS

Q1.

1(a)	One-way stretch followed by a shear.	B2	Both named correctly. Award B1 if given in the wrong order.
		2	
1(b)	$\mathbf{M}^{-1} = \frac{1}{a}\begin{pmatrix}1 & -b \\ 0 & a\end{pmatrix}$	M1 A1	
		2	
1(c)	$a = 2$	B1	
	$\begin{pmatrix}a & b \\ 0 & 1\end{pmatrix}\begin{pmatrix}x \\ y\end{pmatrix} = \begin{pmatrix}ax+by \\ y\end{pmatrix}$	B1	Transforms $\begin{pmatrix}x \\ y\end{pmatrix}$ to $\begin{pmatrix}X \\ Y\end{pmatrix}$.
	$= \begin{pmatrix}ax - \frac{1}{3}bx \\ -\frac{1}{3}x\end{pmatrix}$	M1	Uses $x + 3y = 0$.
	$x = ax - \frac{1}{3}bx \Rightarrow 1 = a - \frac{1}{3}b$	M1	Uses that line is invariant (or $X + 3Y = 0$).
	$b = 3$	A1	
		5	

Q2.

4(a)	Reflection in the line $y = x$.	B1	Only mention reflection (and no other transformation).
		1	
4(b)	Rotation	B1	Writes 'rotation' or 'rotate' (and no other transformation)
	$\frac{1}{3}\pi$ **anticlockwise** about the origin.	B1	Or 60°
		2	
4(c)	$\mathbf{AB} = \begin{pmatrix}\frac{1}{2}\sqrt{3} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2}\sqrt{3}\end{pmatrix}$ or $\det \mathbf{AB} = \det \mathbf{A} \det \mathbf{B}$	M1	Finds **AB** or uses product of determinants. Full marks here may be obtained by arguing that both reflection and rotation preserve [absolute] value of area and so also does their combination.
	$\det \mathbf{AB} = -1$	B1	
	Area of PQR = |-1| Area of DEF	A1	
		3	
4(d)	$(\mathbf{AB})^{-1} = -\begin{pmatrix}-\frac{1}{2}\sqrt{3} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2}\sqrt{3}\end{pmatrix} = \begin{pmatrix}\frac{1}{2}\sqrt{3} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2}\sqrt{3}\end{pmatrix}$	M1 A1	or using $(\mathbf{AB})^{-1} = \mathrm{B}^{-1}\mathrm{A}^{-1}$
		2	

Q3.

4(a)	$\begin{pmatrix}\cos 60 & -\sin 60 \\ \sin 60 & \cos 60\end{pmatrix} = \begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}$	B1	Rotation 60° anticlockwise about the origin.
	$\begin{pmatrix}d & 0 \\ 0 & 1\end{pmatrix}$	B1	One-way stretch in the x-direction, scale factor d.
	$\mathbf{M} = \begin{pmatrix}d & 0 \\ 0 & 1\end{pmatrix}\begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix} = \begin{pmatrix}\frac{d}{2} & -\frac{d\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}$	M1 A1	Correct order.
		4	
4(b)	$d = \frac{1}{2}d^2$	M1	Uses value of $\det \mathbf{M}$.
	$d \neq 0 \Rightarrow d = 2$	A1	AG
		2	

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4(c)	$\mathbf{M}^{-1}=\frac{1}{2}\begin{pmatrix}\frac{1}{2} & \sqrt{3}\\ -\frac{\sqrt{3}}{2} & 1\end{pmatrix}$	B1 FT	Inverse of *their* **M**.
	$\mathbf{N}=\mathbf{M}^{-1}\begin{pmatrix}1 & 1\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}=\frac{1}{2}\begin{pmatrix}\frac{1}{2} & \sqrt{3}\\ -\frac{\sqrt{3}}{2} & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}$	M1	Multiplies on the left by the inverse of *their* **M**.
	$=\frac{1}{4}\begin{pmatrix}1+\sqrt{3} & 1+\sqrt{3}\\ 1-\sqrt{3} & 1-\sqrt{3}\end{pmatrix}$	A1	CAO
		3	
4(d)	$\begin{pmatrix}1 & 1\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x+y\\ \frac{1}{2}x+\frac{1}{2}y\end{pmatrix}$	B1	Transforms $\begin{pmatrix}x\\ y\end{pmatrix}$ to $\begin{pmatrix}X\\ Y\end{pmatrix}$.
	$\frac{1}{2}x+\frac{1}{2}mx=m(x+mx)$	M1 A1	Uses $y=mx$ and $Y=mX$.
	$1+m=2m+2m^2 \Rightarrow 2m^2+m-1=0$	A1	
	$y=\frac{1}{2}x$ and $y=-x$	A1	WWW
		5	

Q4.

4(a)	$\begin{pmatrix}0 & 1 & 1\\ -1 & 2 & 0\end{pmatrix}\begin{pmatrix}2 & k & k\\ 5 & -1 & 3\\ 1 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 1\\ 1 & 0\end{pmatrix}=\begin{pmatrix}0 & 1 & 1\\ -1 & 2 & 0\end{pmatrix}\begin{pmatrix}2+k & k\\ 8 & -1\\ 2 & 0\end{pmatrix}$ Or $\begin{pmatrix}0 & 1 & 1\\ -1 & 2 & 0\end{pmatrix}\begin{pmatrix}2 & k & k\\ 5 & -1 & 3\\ 1 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 1\\ 1 & 0\end{pmatrix}=\begin{pmatrix}6 & -1 & 4\\ 8 & -k-2 & -k+6\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 1\\ 1 & 0\end{pmatrix}$	M1 A1	Multiplies two matrices correctly.
	$\begin{pmatrix}10 & -1\\ -k+14 & -k-2\end{pmatrix}$	A1	
		3	
4(b)	$2\begin{vmatrix}-1 & 3\\ 0 & 1\end{vmatrix}-k\begin{vmatrix}5 & 3\\ 1 & 1\end{vmatrix}+k\begin{vmatrix}5 & -1\\ 1 & 0\end{vmatrix}=0$ leading to $-2-2k+k=0$	M1 A1	Sets determinant equal to zero and forms linear equation.
	$k=-2$	A1	
		3	

4(c)	$\begin{pmatrix}10 & -1\\ 16 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}10x-y\\ 16x\end{pmatrix}$	M1	Transforms $\begin{pmatrix}x\\ y\end{pmatrix}$. Allow $q\begin{pmatrix}1\\ m\end{pmatrix}$ where q is x, t or a nonzero number.
	$10x-mx=X$ and $16x=mX$	M1 A1	Uses $y=mx$ and $Y=mX$. Expect $16x=m(10x-mx)$.
	$16=10m-m^2 \quad [m^2-10m+16=0]$	A1	OE
	$y=2x$ and $y=8x$	A1	
		5	