Momentum 2 MS

Q1.

2(a)	Attempt at use of conservation of momentum	M1	4 terms implied, i.e. m and km included before and after collision. Velocity after collision is the same for m and km.
	$km\times 6 - m\times 2 = (km+m)\times 4$	A1	
	$k=3$	A1	
		3	
2(b)	KE initial = $\frac{1}{2}\times km\times 6^2 + \frac{1}{2}\times m\times(-2)^2$ KE after = $\frac{1}{2}\times(km+m)\times 4^2$	M1	Attempt at any of the three possible KE terms, unsimplified. k need not be substituted here.
	Loss of KE = $24m$ J	A1 FT	KE loss = $56m - 32m$ FT on *their* k, KE loss $=(10k-6)m$, $k>0.6$.
		2	

Q2.

7(a)	For Q: $-2mg\sin\alpha - F = 2ma$ $[-16m - 7.2m = 2ma]$ $R = 2mg\cos\alpha$ $[= 12m]$	M1	Apply Newton's 2nd law along or perpendicular to the plane to particle Q. Must use values for α or sin α or cos α.
		A1	Both correct.
	$F = 0.6\times 2mg\cos\alpha = 0.6\times 0.6\times 20m$ $[= 7.2m]$ $[2(m)a = -2(m)g\,(0.8) - 0.6\times 2(m)g\,(0.6)]$	M1	Using $F=0.6R$ where R is a component of $2mg$ only
	Acceleration of Q up the plane while moving up the plane is $a = -11.6$ ms^{-2}	A1	AG
		4	
7(b)	For P: $mg\sin\alpha - 0.6R = ma$, leading to $8m - 3.6m = ma$ $[R = mg\cos\alpha = 6m,\ a = 4.4\text{ ms}^{-2}]$	M1	Apply Newton's 2nd law to attempt to find the acceleration of particle P. Must use values for α or sin α.
	Q comes to rest when $10 - 11.6T_1 = 0$, $\left[T_1 = \frac{25}{29} = 0.862\right]$	M1	For using constant acceleration equations to attempt to determine when $v_Q = 0$.
	For P $s_{P(\text{down})} = \frac{1}{2}\times 4.4\times T_1^2$ $[=1.635]$ For Q $s_{Q(\text{up})} = 10T_1 + \frac{1}{2}\times(-11.6)\times T_1^2$ $[=4.31]$	M1	Use constant acceleration equations to attempt to find either $s_{P(\text{down})}$ or $s_{Q(\text{up})}$ at time T_1.
	$d = 6.4 - s_{P(down)} - s_{Q(up)}$ $[=0.455]$ and to find T_2 $[=0.12]$ by using $d = s_{P2} - s_{Q2} = (4.4T_1)\times T_2$ $[s_{P2}$ and s_{Q2} are distances travelled by P and Q in time $T_2]$	M1	For attempting to find the extra distance $d[=0.455]$ needed to reach 6.4 m and using $u_P = 4.4T_1$ at T_1 to find T_2 as $d = (4.4T_1)T_2 + \frac{1}{2}\times 4.4T_2^2 - \frac{1}{2}\times 4.4T_2^2$.
	Time before collision = $[t = T_1 + T_2 = 0.862 + 0.12 =]0.982$	A1	$t = 0.98194357\ldots$

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7(b)	**Alternative method for Question 7(b)**		
	For P: $mg\sin\alpha - 0.6R = ma$, leading to $8m - 3.6m = ma$ $[R = mg\cos\alpha = 6m, a = 4.4\text{ ms}^{-2}]$	M1	Apply Newton's 2nd law to attempt to find the acceleration of particle P. Must use values for α or $\sin\alpha$
	Q comes to rest when $10 - 11.6T_1 = 0$, $\left[T_1 = \frac{25}{29} = 0.862\right]$	M1	For using constant acceleration equations to attempt to determine when $v_Q = 0$
	For P $s_{P(\text{down})} = \frac{1}{2} \times 4.4 \times t^2$ For Q $s_{Q(\text{up})} = 10T_1 + \frac{1}{2} \times (-11.6)T_1^2 - \frac{1}{2} \times 4.4(t - T_1)^2$	M1	Use constant acceleration equations to attempt to find either $s_{P(\text{down})}$ or $s_{Q(\text{up})}$ at time t where t is the total time before collision.
	$\frac{1}{2} \times 4.4t^2 + 10T_1 + \frac{1}{2} \times (-11.6)T_1^2 - \frac{1}{2} \times 4.4(t - T_1)^2 = 6.4$	M1	For using $s_{P(\text{down})} + s_{Q(\text{up})} = 6.4$ and solving for t
	Time before collision is $t = 0.982$ s	A1	$t = 0.98194357\ldots$
		5	
	Special case for those who do not take into account the fact that Q comes to rest and then changes its direction		
	For P: $mg\sin\alpha - 0.6R = ma$, leading to $8m - 3.6m = ma$ $[R = mg\cos\alpha = 6m, a = 4.4\text{ ms}^{-2}]$	M1	Apply Newton's 2nd law to attempt to find the acceleration of particle P. Must use values for α or $\sin\alpha$.
	For P $s_{p(\text{down})} = (\pm)\frac{1}{2} \times 4.4t^2$ For Q $s_{q(\text{up})} = (\pm)\ 10t + \frac{1}{2} \times (-11.6)t^2$	M1	For using constant acceleration equations to attempt to find either $s_{p(\text{down})}$ or $s_{q(\text{up})}$.
	$s_p + s_q = 6.4$ leading to $\frac{1}{2} \times 4.4t^2 + 10t + \frac{1}{2} \times (-11.6)t^2 = 6.4$	M1	For applying $(\pm)\ s_p + (\pm)\ s_q = 6.4$ using their expressions for s_p and s_q to set up and solve a 3-term quadratic equation in t to obtain at least 1 solution.

7(b)	Time that particles are in motion before collision $= t = 1$ s	A1	Must reject $t = 16/9$ **Maximum mark 4 out of 5**
		4	
7(c)	$u_{p(\text{down})} = 0 + 4.4 \times 0.982\ [= 4.3208]$	B1 FT	Allow ±4.4. FT on *their* 4.4 and *their* 0.982
	$u_{q(\text{down})} = 4.4 \times 0.12\ [= 0.528]$	B1 FT	Allow ±4.4. FT on *their* 4.4 and *their* 0.12
	$\pm m \times 4.3208 \pm 2m \times 0.528 = \pm(m + 2m)v$ [Correct equation is $m \times 4.3208 + 2m \times 0.528 = \pm(m + 2m)v$]	M1	Apply conservation of momentum, 4 terms, using *their* u_p and u_q values with m and $2m$ respectively. Velocity of P and Q after impact must be equal.
	Speed of combined particle immediately after impact $= v = 1.79\text{ ms}^{-1}$	A1	Must be positive
	Special case for those who do not take into account the fact that Q comes to rest and then changes its direction		
	$u_{p(\text{down})} = 0 + 4.4 \times 1\ [= 4.4]$	B1 FT	Allow ±4.4, FT on *their* 1 and *their* 4.4
	$u_{q(\text{up})} = 10 - 11.6 \times 1\ [= -1.6]$ so $u_{q(\text{down})} = 1.6$	B1 FT	Allow $\pm(10 - 11.6 \times 1)$, FT on *their* 1
	$\pm m \times 4.4 \pm 2m \times 1.6 = \pm(m + 2m)v$	M1	Apply conservation of momentum, 4 terms, using their u_p and u_q values with m and $2m$ respectively. Velocity of P and Q after impact must be equal.
	Speed of combined particle immediately after impact $= v = 2.53\text{ ms}^{-1}$	A1	Allow $v = \frac{38}{15}$. Must be positive.
		4	

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Q3.

1(a)	$120\times8=120v+40v$	M1	Applying conservation of momentum.
	$v=6\ \text{ms}^{-1}$	A1	
		2	
1(b)	$1600-4800=160a$ leading to $a=-20$	M1	Applying Newton's 2nd law to the system.
	$0=6^2+2\times(-20)\times s$	M1	Use of constant acceleration equations such as $v^2=u^2+2as$.
	Distance travelled by post = 0.9 m	A1	
	Alternative method for question 1(b)		
	Initial KE = $\frac{1}{2}\times160\times6^2$	M1	Use of KE = $\frac{1}{2}mv^2$ for combined mass.
	$\frac{1}{2}\times160\times6^2+160\times10\times s=4800s$	M1	Forms work/energy equation.
	Distance travelled by post = 0.9 m	A1	
		3	

Q4.

7(a)	**Alternative scheme for question 7(a)**		
	Attempt PE loss or KE gain	M1	Use of either PE = mgh or KE = ½ mv^2
	PE loss $=0.1\times g\times0.45\sin16.3=0.1\times g\times0.45\times\frac{7}{25}$ $\left[=\frac{63}{500}=0.126\right]$ KE gain $=\frac{1}{2}\times0.1\times0.6^2$ $\left[=\frac{9}{500}=0.018\right]$	A1	Both correct.
	$R=0.1g\times\cos\alpha=0.1g\times\frac{24}{25}=0.1g\times\cos16.3°$ $\left[R=\frac{24}{25}=0.96\right]$	B1	Must use a value for $\cos\alpha$.
	$0.1\times g\times0.45\times\frac{7}{25}=\frac{1}{2}\times0.1\times0.6^2+F\times0.45$ $\left[\frac{63}{500}=\frac{9}{500}+\mu\times\frac{54}{125}\right]$ or $[0.126=0.018+\mu\times0.432]$	M1	Use of work-energy equation as PE loss = KE gain + WD against friction
	$F=\mu\times0.1g\times\frac{24}{25}$ $\left[F=\frac{24\mu}{25}=0.96\mu\right]$	M1	Use of $F=\mu R$, where R is a component of $0.1g$
	$\mu=0.25$	A1	**AG** Must be from exact working $\mu=0.25$ only
		6	

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Q5.

7(a)	$0.4\times 3+0.2\times 2=0.4\times 2.5+0.2v$	M1	Use of conservation of momentum with 4 terms. Allow sign errors.
	$v=3\ \text{ms}^{-1}$	A1	Allow M1A0 if g included with the masses.
		2	
7(b)	For A $\pm 0.4g\sin 30°=0.4a$ or for B $\pm 0.2g\sin 30°=0.2a$ or $\pm mg\sin 30°=ma$	M1	For either. Allow sin/cos mix.
	$a=\pm 5$ or $\pm g\sin 30°$	A1	Allow $g\sin 30°$ without working for M1A1
	For B when hits barrier $v^2=3^2+2\times 5\times 1.6$ $[\Rightarrow v=5]$ OR $v=u+at\Rightarrow v=3+5\times 0.4\ [\Rightarrow v=5]$	M1	Using *their* $a\neq\pm g$ and *their* v from part **(a)** OR: use of $s=\frac{u+v}{2}t$ $1.6=\frac{3+v}{2}\times 0.4[\Rightarrow v=5]$ OR $\frac{1}{2}\times 0.2\times v^2-\frac{1}{2}\times 0.2\times 3^2=0.2\times 1.6\times g\sin 30$
	Speed after hitting barrier $=0.1\times 5=0.5$	A1	**AG**
	$v_A=2.5+5\times 0.44\ [=4.7]$ $v_B=-0.5+5\times 0.04\ [=-0.3]$ or $v_B=0.5+(-5)\times 0.04\ [=0.3]$	*M1	Use of $v=u+at$ for either with correct t-value, with initial speeds ± 2.5 or ± 0.5 *their* $\pm a\neq\pm g$
	$0.4\times 4.7+0.2\times(-0.3)=0.6\,v_{comb}$	DM1	Use of $v=u+at$ for BOTH with correct t-values, initial speeds ± 2.5, ± 0.5 and $\pm$ *their* acceleration (same for both) and use of conservation of momentum with correct number of terms. Allow sign errors.
	$v_{comb}=3.03\ \text{ms}^{-1}$	A1	Allow $v=\frac{91}{30}=3\frac{1}{30}$ Allow DM1A0 if g included with the masses.
		7	

Q6.

1(a)	Conservation of momentum	M1	3 terms; allow M1 if speed of A after collision is $\frac{1}{4}\times 8.5$. Allow $5\times 8.5=5X+3Y$ where $\lvert X\rvert$ and $\lvert Y\rvert$ are different which may be seen by later work. If $\lvert X\rvert$ and $\lvert Y\rvert$ are subsequently used as being equal then M0.
	$5\times 8.5=5\times 0.25v+3v$	A1	OE e.g. $5\times 8.5=5V+3\times 4V$
	Speed of B $=10\text{ms}^{-1}$	A1	Do not award if 10 from using mgv, maximum 2/3 -10 is A0 as speed required not velocity
		3	
1(b)	KE before $=\frac{1}{2}\times 5\times 8.5^2\,[=180.625]$ KE after $=\frac{1}{2}\times 5\times 2.5^2+\frac{1}{2}\times 3\times 10^2\,[=15.625+150=165.625]$	1	Attempt at any of the 3 terms for KE, using their 10ms^{-1} Not $\frac{1}{2}\times(5+3)\times 8.5^2$, not $\frac{1}{2}\times(5+3)\times 2.5^2$ not $\frac{1}{2}\times(5+3)\times 10^2$ unless $\lvert X\rvert=\lvert Y\rvert$ seen
	KE loss $[=180.625-165.625]=15$ J	A1	Accept -15, AWRT ± 15.0
		2	

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Q7.

1(a)	$0.3\times4+0=0.3v+0.2\times3$	M1	For attempt at use of conservation of momentum
	Speed = 2 ms^{-1}	A1	
		2	
1(b)	$0.2\times3+0=(0.2+m)\times2$	M1	For attempt at use of conservation of momentum
	$m=0.1$	A1	
		2	

Q8.

2	Use conservation of momentum $6\times5+2\times(-3)=6v_A+2v_B$	*M1	4 dimensionally correct terms. Allow sign errors, v_A and v_B must be different.
	Use $v_B=v_A+2$ or $v_A=v_B-2$ with their momentum equation and solve for v_A or v_B	DM1	Allow $v_B=v_A\pm2$ or $v_A=v_B\pm2$.
	$v_A=2.5$ or $v_B=4.5$	A1	
	Attempt at initial KE, or final KE, or change in KE for A, or change in KE for B Initial KE $=\frac{1}{2}\times6\times5^2+\frac{1}{2}\times2\times(-3)^2[=84]$ Final KE $=\frac{1}{2}\times6\times(their2.5)^2+\frac{1}{2}\times2\times(their4.5)^2$ Change in KE for $A=\pm\left(\frac{1}{2}\times6\times5^2-\frac{1}{2}\times6\times(their2.5)^2\right)$ Change in KE for $B=\pm\left(\frac{1}{2}\times2\times(-3)^2-\frac{1}{2}\times2\times(their4.5)^2\right)$	M1	Allow use or their v_A and/or v_B. Allow if 2 KE equations seen.
	Loss of KE = 45J	A1	Allow –45J. Allow if mgv used in momentum equation.
		5	

Q9.

6(a)	$0.3\times2[+0]=0.3\times0.6+0.4\times v$	M1	For use of conservation of momentum. Must be 3 terms. Allow sign errors.
	Speed of B = 1.05 ms^{-1}	A1	AG Allow M1 A0 if g included with the masses.
		2	
6(b)	$0.4\times1.05[+0]=(0.4+m)\times0.5$	M1	For use of conservation of momentum. Must be 3 terms. Allow sign errors.
	$m=0.44$ or $\frac{11}{25}$	A1	Allow M1 A0 if g included with the masses.
		2	

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6(c)	1.2[m] or 0.9[m]	**B1**	Must be a distance as some candidates get 1.2 from $\frac{0.6}{0.5}$.
	$0.5t$	**B1**	Seen but not $+\frac{1}{2}at^2$ unless later state that $a=0$. B0 if only $0.5t=2.1$ (may see solving to find $t=3.5$).
	$0.6t$	**B1**	Seen but not $+\frac{1}{2}at^2$ unless later state that $a=0$. Allow B2 in place of second and third B1 marks for 'difference in speeds is 0.1 [ms^{-1}]'.
	Distances equal so $0.6t-0.9=0.5t$ and solve for t Or $t=\frac{0.9}{0.6-0.5}$	**M1**	OE Must get to '$t=$'. Allow $\pm$ *their* 0.9 but not ±1.2 or ±2.1 or ±1.5. Do not allow $0.6t+0.5t=\pm0.9$. Do not allow M1 if either or both terms include $+\frac{1}{2}at^2$ unless they state $a=0$.
	Time = 9s	**A1**	CWO

6(c)	**Alternative method for question 6(c) using time from start of motion**		
	1[m] or 1.1[m]	**B1**	
	$0.6T$	**B1**	Seen but not $+\frac{1}{2}aT^2$ unless later state that $a=0$.
	$0.5T$	**B1**	Seen but not $+\frac{1}{2}aT^2$ unless later state that $a=0$. Allow **B2** in place of second and third B1 marks for 'difference in speeds is 0.1 [ms^{-1}]'.
	Distances equal so $0.6T-1.1=0.5T$ and solve for T Or $T=\frac{1.1}{0.6-0.5}$	**M1**	OE Must get to '$T=$'. Allow $\pm$ *their* 1.1 but not ±1.0 or ±2.1. Do not allow $0.6T+0.5T=\pm1.1$. Do not allow M1 if either or both terms include $+\frac{1}{2}aT^2$ unless they state $a=0$
	$\Rightarrow$ Time from BC collision = $11-2=9$ s	**A1**	CWO

6(c)	**Alternative method for question 6(c) using distance travelled from time when *B* and *C* collide**		
	1.2[m] or 0.9[m]	**B1**	
	Time taken for A is $\frac{d+0.9}{0.6}$ Or $d+0.9=0.6t$	**B1 FT**	Allow $\pm$ *their* 0.9 but not ±1.2 or ±2.1 or ±1.5.
	Time taken for BC is $\frac{d}{0.5}$ Or $d=0.5t$	**B1**	
	$\frac{d+0.9}{0.6}=\frac{d}{0.5}\Rightarrow d=4.5\Rightarrow \text{Time}=\frac{4.5}{0.5}$	**M1**	Must get to '$t=$'. Allow $\pm$ *their* 0.9 but not ±1.2 or ±2.1.
	Time = 9s	**A1**	CWO
		5	