Probability Generating Functions 1

Q1.

Aisha has a bag containing 3 red balls and 3 white balls. She selects a ball at random, notes its colour and returns it to the bag; the same process is repeated twice more. The number of red balls selected by Aisha is denoted by X.

(a) Find the probability generating function $G_X(t)$ of X. [2]

Basant also has a bag containing 3 red balls and 3 white balls. He selects three balls at random, without replacement, from his bag. The number of red balls selected by Basant is denoted by Y.

(b) Find the probability generating function $G_Y(t)$ of Y. [3]

The random variable Z is the total number of red balls selected by Aisha and Basant.

(c) Find the probability generating function of Z, expressing your answer as a polynomial. [3]

(d) Use the probability generating function of Z to find $E(Z)$ and $Var(Z)$. [5]

Q2.

A bag contains 4 red balls and 6 blue balls. Rassa selects two balls at random, without replacement, from the bag. The number of red balls selected by Rassa is denoted by X.

(a) Find the probability generating function, $G_X(t)$, of X. [2]

Rassa also tosses two coins. One coin is biased so that the probability of a head is $\frac{2}{3}$. The other coin is biased so that the probability of a head is p. The probability generating function of Y, the number of heads obtained by Rassa, is $G_Y(t)$. The coefficient of t in $G_Y(t)$ is $\frac{7}{12}$.

(b) Find $G_Y(t)$. [3]

The random variable Z is the sum of the number of red balls selected and the number of heads obtained by Rassa.

(c) Find the probability generating function of Z, expressing your answer as a polynomial. [3]

(d) Use the probability generating function of Z to find $E(Z)$. [2]

Probability Generating Functions 1

Q3.

The discrete random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = 0.2t + 0.5t^2 + 0.3t^3.$$

The random variable Y is the sum of two independent observations of X.

(a) Find the probability generating function of Y, giving your answer as an expanded polynomial in t.

[3]

(b) Use the probability generating function of Y to find E(Y) and Var(Y).

[5]

_____

Q4.

Keira has two unbiased coins. She tosses both coins. The number of heads obtained by Keira is denoted by X.

(a) Find the probability generating function $G_X(t)$ of X.

[1]

Hassan has three coins, two of which are biased so that the probability of obtaining a head when the coin is tossed is $\frac{1}{3}$. The corresponding probability for the third coin is $\frac{1}{4}$. The number of heads obtained by Hassan when he tosses these three coins is denoted by Y.

(b) Find the probability generating function $G_Y(t)$ of Y.

[3]

The random variable Z is the total number of heads obtained by Keira and Hassan.

(c) Find the probability generating function of Z, expressing your answer as a polynomial.

[3]

(d) Use the probability generating function of Z to find E(Z).

[2]

_____

Q5.

The random variable X has the binomial distribution B(n, p).

(a) Write down an expression for P$(X = r)$ and hence show that the probability generating function of X is $(q + pt)^n$, where $q = 1 - p$.

[3]

(b) Use the probability generating function of X to prove that E$(X) = np$ and Var$(X) = np(1 - p)$.

[5]

_____

Probability Generating Functions 1

Q6.

Tanji has a bag containing 4 red balls and 2 blue balls. He selects 3 balls at random from the bag, without replacement. The number of red balls selected by Tanji is denoted by X.

(a) Find the probability generating function $G_X(t)$ of X. [2]

Tanji also has two coins, each biased so that the probability of obtaining a head when it is thrown is $\frac{1}{4}$. He throws the two coins at the same time. The number of heads obtained is denoted by Y.

(b) Find the probability generating function $G_Y(t)$ of Y. [2]

The random variable Z is the sum of the number of red balls selected by Tanji and the number of heads obtained.

(c) Find the probability generating function of Z, expressing your answer as a polynomial. [3]

(d) Use the probability generating function of Z to find $E(Z)$ and Var (Z). [5]

_____