Probability Generating Functions 2 MS

Q1.

4(a)	$G'(t) = \frac{4t}{(3-2t^2)^2}$	M1	Differentiate to obtain $\frac{kt}{(3-2t^2)^2}$ OE.
	so E(X) = G'(1) = 4	A1	CAO WWW
	$G''(t) = \frac{12+24t^2}{(3-2t^2)^3}$ or $4(3-2t^2)^{-2} + 32t^2(3-2t^2)^{-3}$	M1	OE. Differentiate, allow only numerical or sign slips.
	$\text{Var}(X) = G''(1) + G'(1) - (G'(1))^2 = 36 + 4 - 16$	M1	Substitute their values into correct formula, dependent on attempt at $G''(t)$.
	[Var(X)] = 24	A1	CAO WWW
		5	
4(b)	$G_X(t) = \frac{1}{3-2t^2} = (3-2t^2)^{-1}$ $= \frac{1}{3}\left(1 + \frac{2}{3}t^2 + \frac{4}{9}t^4 + \ldots\right)$	M1	Expand given expression or give expression for the term in t^4.
	P(X = 4) = their coefficient of t^4	M1	Found from legitimate method.
	[P(X = 4) =] $\frac{4}{27}$	A1	WWW
		3	

Q2.

5(a)	$P(3, 6, 9) = \frac{3}{9}\times\frac{2}{8}\times\frac{1}{7} = \frac{6}{504}$ $P(\text{Two of } 3, 6, 9) = \frac{3}{9}\times\frac{2}{8}\times\frac{6}{7}\times 3 = \frac{108}{504}$ $P(\text{one of } 3, 6, 9) = \frac{3}{9}\times\frac{6}{8}\times\frac{5}{7}\times 3 = \frac{270}{504}$ $P(\text{none of } 3, 6, 9) = \frac{6}{9}\times\frac{5}{8}\times\frac{4}{7} = \frac{120}{504}$	B1	At least 2 probabilities correct.
	$G_X(t) = \frac{20}{84} + \frac{45}{84}t + \frac{18}{84}t^2 + \frac{1}{84}t^3$	M1 A1	Attempt with at least 3 probabilities in a polynomial, CAO.
		3	
5(b)	$P(\text{both even}) = \frac{12}{72}$ $P(\text{one even}) = \frac{40}{72}$ $P(\text{no even}) = \frac{20}{72}$	M1	
	$G_Y(t) = \frac{5}{18} + \frac{10}{18}t + \frac{3}{18}t^2$	A1	CAO
		2	

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Q3.

4	**(i)**		G'Y(t)= $0.18t+0.72t^2+1.36t^3+1.2t^4+0.54t^5$ Sub t=1,(0.18+0.72+1.36+1.2+0.54)	M1	≥3 terms correct. M1 for diffn and sub t=1	y 2 3 4 5 6 p 0.09 0.24 0.34 0.24 0.09 B1 E(Y)=0.18+….=4 B1 E(Y2)=0.36+…..=17.2 M1 Var (Y)=17.2-42=1.2 M1A1
			4 G''Y(t)=$0.18+1.44t+4.08t^2+4.8t^3+2.7t^4$ Sub t=1 and use correct formula for Var (0.18+1.44+4.08+4.8+2.7+"4"-"16") 1.2	A1 M1 M1 A1 **[5]**	≥3 terms correct ft	
	(ii)		Attempt to factorise into ($0.3t$+…)($0.3t$+…) $0.3t+0.4t^2+0.3t^3$	M1M1 A1 **[3]**	Attempt to find √(GY(t)) seen or implied.	Or $0.3t^3$ twice.
	(iii)		0.4	B1ft [1]	Allow non-numerical answer, but coeff of t2 is not enough.	

Q4.

5(a)	$(1+4+9+16)k=1$ so $k=\frac{1}{30}$	**B1**	
		1	
5(b)	$G_X(t)=\frac{1}{30}t+\frac{4}{30}t^2+\frac{9}{30}t^3+\frac{16}{30}t^4$	**M1A1**	Using their k in a polynomial, at least two terms correct for their k.
		2	
5(c)	$\left(\frac{1}{30}t+\frac{4}{30}t^2+\frac{9}{30}t^3+\frac{16}{30}t^4\right)\left(\frac{1}{4}+\frac{1}{2}t+\frac{1}{4}t^2\right)$	**M1**	Method and attempt to multiply.
	$\frac{1}{120}\left(t+6t^2+18t^3+38t^4+41t^5+16t^6\right)$	**M1A1**	Multiplication to obtain single polynomial of order 6.
		3	
5(d)	Given: G'(1) = 13/3 G"(t) = $\frac{1}{120}(12+108t+456t^2+820t^3+480t^4)$	**M1**	Differentiate twice.
	Var(X) = G"(1) + $\frac{13}{3}-\left(\frac{13}{3}\right)^2$	**M1**	Use correct formula.
	$\frac{1876}{120}+\frac{13}{3}-\frac{169}{9}=\frac{107}{90}$ or 1.19	**A1**	CAO
		3	

Q5.

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6	(i)		P(0) = 0.16, P(1) = 0.48, P(2) = 0.36 $G_Y(t) = 0.16 + 0.48t + 0.36t^2$	B1 M1A1 **[3]**	
	(ii)		$G_X(t) \times G_Y(t)$ soi $0.02 + 0.12t + 0.285t^2 + 0.335t^3 + 0.195t^4 + 0.045t^5$	M1 A1A1 **[3]**	At least 4 terms correct; All correct.
	(iii)		$E(Z) = G_Z'(1)$ $[=0.12+0.57t+1.005t^2+0.78t^3+0.225t^4]$ Sub $t = 1$ 2.7 Attempt 2nd derivative of G_Z Attempt $G''(1) + G'(1) - G'(1))^2$ $(G''(1) = 5.82)$ 1.23 Alternative methods. 3x0.5 or 2x0.6; added ; 2.7 M1M1A1 3x0.5x0.5 or 2x0.6x0.4; added; 1.23 M1M1A1 P(Z=0)=0.02 etc B1 ;E(Z)=Σzp=2.7 M1A1 $E(Z^2)=\Sigma z^2p=(8.52)$ M1;-2.7^2 M1 1.23A1	M1 M1dep A1 M1 M1dep A1 **[6]**	Differentiate. -ve var, M0
	(iv)		0.335	B1ft **[1]**	Coeff t^3 from (ii)

Q6.

1 (i)	$\sum_{x=0}^{n} \binom{n}{x} p^x q^{n-x} t^x$	M1	From $E(t^X)$
	$= \sum_{x=0}^{n} \binom{n}{x} (pt)^x q^{n-x}$	A1 **2**	M1A0 $\sum$ without limits $G_X(t)$=q+pt M1 then argument A0
(ii)	$G_T(t) = (q+pt)^n(q+pt)^{2n}$ $= (q+pt)^{3n}$ So $T \sim B(3n, p)$	M1 A1 M1 A1 **4** **[6]**	Multiplying pgfs For B For parameters