# Probability Generating Functions 2

Q1.

$X$ is a discrete random variable which takes the values 0, 2, 4, … . The probability generating function of $X$ is given by

$$G_X(t) = \frac{1}{3-2t^2}.$$

**(a)** Find $E(X)$ and $Var(X)$. [5]

**(b)** Find $P(X = 4)$. [3]

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Q2.

Nine balls labelled 1, 2, 3, 4, 5, 6, 7, 8, 9 are placed in a bag. Kai selects three balls at random from the bag, without replacement. The random variable $X$ is the number of balls selected by Kai that are labelled with a multiple of 3.

**(a)** Find the probability generating function $G_X(t)$ of $X$. [3]

The balls are replaced in the bag.

Jacob now selects two balls at random from the bag, without replacement. The random variable $Y$ is the number of balls selected by Jacob that are labelled with an even number.

**(b)** Find the probability generating function $G_Y(t)$ of $Y$. [2]

The random variable $Z$ is the sum of the number of balls that are labelled with a multiple of 3 selected by Kai and the number of balls that are labelled with an even number selected by Jacob.

**(c)** Find the probability generating function of $Z$, expressing your answer as a polynomial. [3]

**(d)** Use the probability generating function of $Z$ to find $E(Z)$. [2]

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Q3.

The discrete random variable $Y$ has probability generating function

$$G_Y(t) = 0.09t^2 + 0.24t^3 + 0.34t^4 + 0.24t^5 + 0.09t^6.$$

**(i)** Find the mean and variance of $Y$. **[5]**

$Y$ is the sum of two independent observations of a random variable $X$.

**(ii)** Find the probability generating function of $X$, expressing your answer as a cubic polynomial in $t$. **[3]**

**(iii)** Write down the value of $P(X = 2)$. **[1]**

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Q4.

The random variable $X$ is such that $\mathrm{P}(X = r) = kr^2$ for $r = 1,2,3,4$, where $k$ is a constant.

**(a)** Find the value of $k$. [1]

**(b)** Find the probability generating function $\mathrm{G}_X(t)$ of $X$. [2]

The random variable $Y$ has probability generating function $\mathrm{G}_Y(t) = \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}t^2$.

The random variable $Z$ is the sum of $X$ and $Y$.

**(c)** Assuming that $X$ and $Y$ are independent, find the probability generating function $\mathrm{G}_Z(t)$ of $Z$ as a polynomial in $t$. [3]

**(d)** Given that $\mathrm{E}(Z) = \frac{13}{3}$, use $\mathrm{G}_Z(t)$ to find $\mathrm{Var}(Z)$. [3]

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Q5.

Andrew has five coins. Three of them are unbiased. The other two are biased such that the probability of obtaining a head when one of them is tossed is $\frac{3}{5}$.

Andrew tosses all five coins. It is given that the probability generating function of $X$, the number of heads obtained on the unbiased coins, is $\mathrm{G}_X(t)$, where

$$\mathrm{G}_X(t) = \frac{1}{8} + \frac{3}{8}t + \frac{3}{8}t^2 + \frac{1}{8}t^3.$$

**(i)** Find $G_Y(\mathrm{t})$, the probability generating function of $Y$, the number of heads on the biased coins. **[3]**

**(ii)** The random variable $Z$ is the total number of heads obtained when Andrew tosses all five coins. Find the probability generating function of $Z$, giving your answer as a polynomial. **[3]**

**(iii)** Find $\mathrm{E}(Z)$ and $\mathrm{Var}(Z)$. **[6]**

**(iv)** Write down the value of $\mathrm{P}(Z = 3)$. **[1]**

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Q6.

The random variable $X$ has the distribution $\mathrm{B}(n, p)$.

**(i)** Show, from the definition, that the probability generating function of $X$ is $(q + pt)^n$, where $q = 1 - p$. **[2]**

**(ii)** The independent random variable $Y$ has the distribution $\mathrm{B}(2n, p)$ and $T = X + Y$. Use probability generating functions to determine the distribution of $T$, giving its parameters. **[4]**

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