Poisson Distribution 1 MS

Q1.

3	(i)	Constant average rate of goals scored Goals random Goals indep	B1 B1 [2]	Any two given in context (SR score B1 for any two not in context) Not Goals scored singly (because this is inherent in the context so it's not a condition)
	(ii)	$e^{-1.8}(\frac{1.8^3}{3!}+\frac{1.8^4}{4!}+\frac{1.8^5}{5!})$ $= 0.259$	M1 A1 [2]	Poisson probs, $\lambda = 1.8$. Allow 2, 6 included
	(iii)	$1 - e^{-1.8}$ $(1 - e^{-1.8})^{10}$ $= 0.164$	M1 M1 A1 [3]	Any λ. Allow end errors.
			[Total: 7]	

Q2.

5	(i)	Po(3.3) $e^{-3.3}(1 + 3.3 + \frac{3.3^2}{2})$ $= 0.359$	B1 M1 A1 [3]	seen or implied Poisson P(0) + P(1) + P(2). Allow + P(3) Allow wrong λ. Accept equiv method.
	(ii)	X~Po(36) X~N(36, 36) $\frac{48.5 - 36}{\sqrt{36}}$ $= 2.08(3)$ comp with 1.96 Evidence to support claim	B1 B1 M1 A1 M1 A1√ [6]	 Allow with no or wrong cc or no √ 2.08(3) or 0.0186/0.0187 if area comparison Valid comparison Correct conclusion (ft their z)
			[Total: 9]	

Q3.

4	(i)	Po(4) $1 - e^{-4}(1 + 4 + \frac{4^2}{2!}+\frac{4^3}{3!})$ $= 1 - 0.43347..$ $= 0.567$ or 0.566		M1 A1 M1 A1 [4]	Use of Poisson, any mean Correct mean Allow one end error SC1: $\frac{3.5-4}{\sqrt{3.9984}}$ B1 SC2: Correct Bin method M1 ans 0.567 or 0.566 A1
	(ii)	$\lambda = {}^{n}/_{2500}$ $e^{-\frac{n}{2500}} < 0.01$ $-\frac{n}{2500} < \ln 0.01$ $n > 11512.9\ldots$ Smallest $n = 11513$	$(\frac{2499}{2500})^n$ $(\frac{2499}{2500})^n < 0.01$ $n \times \ln(\frac{2499}{2500}) < \ln 0.01$ $n > 11510.6\ldots$ Smallest $n = 11511$	B1 M1 A1 [3]	 Correct exp'n < 0.01. Allow ' =' Allow by trial
				[Total: 7]	

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Q4.

1	**(i)**	Mean = 2.6	B1		
		Var = 4 × 1.3	M1		M1 for either 4 ×, or for Var(X) = 1.3 implied
		= 5.2	A1	[3]	
	(ii)	Var ≠ mean or 2X does not take all integer values	B1	[1]	X and X are not independent oe

Q5.

6	**(i)**	Customers arrive independently or randomly	B1	[1]	In context. Allow "singly"
	(ii)	$e^{-6} \times \frac{6^5}{5!}$	M1		Poisson P(5), allow any mean
		= 0.161 (3 sfs)	A1	[2]	
	(iii)	$\lambda = 2.4$	B1		
		$e^{-2}\left(1 + 2.4 + \frac{2.4^2}{2!}\right)$	M1		Poisson P(0, 1, 2), allow their mean allow one end error
		= 0.570 (3 sfs)	A1	[3]	
	(iv)	N(24, 24)	B1		Stated or implied
		$\frac{295 - 24}{\sqrt{24}}$ (= 1.123)	M1		Allow with wrong or no cc and/or no √ Correct area
		Φ("1.123")	M1		
		= 0.869 (3 sfs)	A1	[4]	

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Q6.

7 (i) (a)	$1 - e^{-1.2}(1 + 1.2)$ (= 0.3374) $1 - e^{-1.4}$ (= 0.7534)	M1 A1		M1 for Poisson either P(0 or 1) or P(0) with $\lambda = 1.2$ or 2.4 or 1.4 or 2.8, accept one end error Both expressions fully correct
	$(1 - e^{-1.2}(1 + 1.2)) \times (1 - e^{-1.4})$	M1		Their Poisson P(0 or 1) × P(0)
	= 0.254 (3 sfs)	A1	[4]	
(i) (b)	$\lambda = 2.6$ seen	B1		
	$1 - e^{-2.6}(1 + 2.6 + 2.6^2 \div 2)$	M1		Poisson 1 – P(0, 1, 2), allow 1 – P(0, 1, 2, 3), with attempt at combined λ for M and W. Accept combination method: at least 4 correct terms and "1 –" M1; all terms correct B1
	= 0.482 (3 sfs)	A1	[3]	

Q7.

5 (i) (a)	$P(X \geq 3) = 1 - e^{-3.2}\left(1 + 3.2 + \frac{3.2^2}{2!}\right)$ = 0.62(0) (3 sf)	M1 A1	[2]	Allow one end error
(b)	$P(X = 3) = e^{-3.2}\left(\frac{3.2^3}{3}\right)$ (= 0.22262)	M1		May be implied
	$\left(\frac{P(X=3 \cap X \geq 3)}{P(X \geq 3)} = \frac{P(X=3)}{P(X \geq 3)}\right)$ $= \frac{'0.22262'}{'0.62010'}$	M1		$\frac{\text{Their } P(X=3)}{\text{Their } P(X \geq 3)}$
	= 0.359 (3 sf)	A1	[3]	
(ii) (a)	(Approx) normal with mean 3.2	B1		
	variance = $\frac{3.2}{120}$ or $\frac{2}{75}$ or 0.0267 (3 sfs) oe	B1	[2]	or sd = $\sqrt{\frac{3.2}{120}}$ or 0.163 (3 sfs) oe
(b)	$\frac{3.3-3.2}{\sqrt{\frac{3.2}{120}}}$ (= 0.612)	M1		Allow with cc attempted
	Φ ("0.612") = 0.730 (3 sfs)	M1 A1	[3]	Accept 0.73

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Q8.

7	(i)	$\lambda = 5$ $1 - e^{-5}\left(1 + 5 + \frac{5^2}{2!}\right)$ $= 0.875$	B1 M1 A1 [3]	Any λ. Allow one end error
	(ii)	$X \sim N(120, 120)$ $\frac{109.5-120}{\sqrt{120}}$ $(= -0.9585)$ $1 - \Phi(\text{“}0.9585\text{”})$ $(= 1 - 0.8312)$ $\text{“}0.1688\text{”}^2$ $= 0.0285$ to 0.0286	B1 M1 M1 A1 [4]	May be implied Allow with wrong or no cc or no √
	(iii)	$\lambda = 15 \times \frac{5}{60} + 0.5$ $= 1.75$ $e^{-1.75}\left(\frac{1.75^3}{3!} + \frac{1.75^4}{4!} + \frac{1.75^5}{5!}\right)$ $= 0.247$ (3 sfs)	M1 A1 M1 A1 [4]	Any λ. Allow one end error

Q9.

7	(i)	$\lambda = 4.8$ $E^{-4.8}(1 + 4.8 + \frac{4.8^2}{2!} + \frac{4.8^3}{3!})$ $= 0.294$ (3 sfs)	B1 M1 A1 [3]	P($R = 0, 1, 2$ or 3), their λ allow one end error
	(ii)	$e^{-\lambda} \times \frac{\lambda^4}{4!} = \frac{16}{3} e^{-\lambda} \times \frac{\lambda^2}{2!}$ or without $e^{-\lambda}$ $\frac{\lambda^2}{12} = \frac{16}{3}$ or better $(\lambda = 8)$ $\lambda = 1.6n$ seen or implied $n =$ ‘8’ ÷ 1.6 $= 5$	M1 A1 B1 A1 [4]	$\lambda = 1.6n$ seen or implied B1 $e^{-1.6n} \times \frac{(1.6n)^4}{4!} = \frac{16}{3} e^{1.6n} \times \frac{(1.6n)^2}{2!}$ M1 $\frac{(1.6n)^2}{12} = \frac{16}{3}$ or better A1 $(1.6n = 8)$ $n = 5$ A1
	(iii)	$T \sim N(64, 64)$ $\frac{75.5-64}{\sqrt{64}}$ $(= 1.4375)$ $1 - \Phi(\text{‘}1.4375\text{’})$ $(= 1 - 0.9247)$ $= 0.0753$ to 0.0754	B1 M1 M1 A1 [4]	May be implied Allow with wrong or no cc. No sd/var mixes Finding correct area consistent with their working

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Q10.

5	**(i)**	B(520, 0.008) Po(4.16) $n = 500$ which is large, $np = 4.16$ which is < 5 or p small <0.1	B1 B1B1 B1 [4]	 Po: B1, $\lambda = 4.16$: B1 Both needed
	(ii)	**(a)** $1 - e^{-4.16}(1 + 4.16 + \frac{4.16^2}{2} + \frac{4.16^3}{3!})$ $= 0.597$ (3 sf)	 M1 A1 [2]	 1–P(0,1,2,3) any λ allow one end error
		(b) $e^{-4.16} \times \frac{4.16^n}{n!} > e^{-4.16} \times \frac{4.16^{n+1}}{(n+1)!}$ $1 > \frac{4.16}{n+1}$ $n > 3.16$ Smallest n is 4	M1 A1 A1 [3]	any λ or equiv equn without e and without factorials (Calculation of P(0), P(1),....P(5) scores M1for at least 3 attempted, A1 all correct, A1 for n = 4)