Poisson Distribution 1

Q1.

The number of goals scored per match by Everly Rovers is represented by the random variable X which has mean 1.8.

(i) State two conditions for X to be modelled by a Poisson distribution. [2]

Assume now that $X \sim \text{Po}(1.8)$.

(ii) Find $P(2 < X < 6)$. [2]

(iii) The manager promises the team a bonus if they score at least 1 goal in each of the next 10 matches. Find the probability that they win the bonus. [3]

Q2.

The number of adult customers arriving in a shop during a 5-minute period is modelled by a random variable with distribution Po(6). The number of child customers arriving in the same shop during a 10-minute period is modelled by an independent random variable with distribution Po(4.5).

(i) Find the probability that during a randomly chosen 2-minute period, the total number of adult and child customers who arrive in the shop is less than 3. [3]

(ii) During a sale, the manager claims that more adult customers are arriving than usual. In a randomly selected 30-minute period during the sale, 49 adult customers arrive. Test the manager's claim at the 2.5% significance level. [6]

Q3.

On average, 1 in 2500 people have a particular gene.

(i) Use a suitable approximation to find the probability that, in a random sample of 10 000 people, more than 3 people have this gene. [4]

(ii) The probability that, in a random sample of n people, none of them has the gene is less than 0.01. Find the smallest possible value of n. [3]

Q4.

The random variable X has the distribution Po(1.3). The random variable Y is defined by $Y = 2X$.

(i) Find the mean and variance of Y. [3]

(ii) Give a reason why the variable Y does not have a Poisson distribution. [1]

Poisson Distribution 1

Q5.

Customers arrive at an enquiry desk at a constant average rate of 1 every 5 minutes.

(i) State one condition for the number of customers arriving in a given period to be modelled by a Poisson distribution. [1]

Assume now that a Poisson distribution is a suitable model.

(ii) Find the probability that exactly 5 customers will arrive during a randomly chosen 30-minute period. [2]

(iii) Find the probability that fewer than 3 customers will arrive during a randomly chosen 12-minute period. [3]

(iv) Find an estimate of the probability that fewer than 30 customers will arrive during a randomly chosen 2-hour period. [4]

Q6.

The numbers of men and women who visit a clinic each hour are independent Poisson variables with means 2.4 and 2.8 respectively.

(i) Find the probability that, in a half-hour period,

(a) 2 or more men and 1 or more women will visit the clinic, [4]

(b) a total of 3 or more people will visit the clinic. [3]

(ii) Find the probability that, in a 10-hour period, a total of more than 60 people will visit the clinic. [4]

Q7.

A random variable X has the distribution Po(3.2).

(i) A random value of X is found.

(a) Find $P(X \geqslant 3)$. [2]

(b) Find the probability that $X = 3$ given that $X \geqslant 3$. [3]

(ii) Random samples of 120 values of X are taken.

(a) Describe fully the distribution of the sample mean. [2]

(b) Find the probability that the mean of a random sample of size 120 is less than 3.3. [3]

Poisson Distribution 1

Q8.

At work Jerry receives emails randomly at a constant average rate of 15 emails per hour.

(i) Find the probability that Jerry receives more than 2 emails during a 20-minute period at work. [3]

(ii) Jerry's working day is 8 hours long. Find the probability that Jerry receives fewer than 110 emails per day on each of 2 working days. [4]

(iii) At work Jerry also receives texts randomly and independently at a constant average rate of 1 text every 10 minutes. Find the probability that the total number of emails and texts that Jerry receives during a 5-minute period at work is more than 2 and less than 6. [4]

Q9.

A random variable X has the distribution Po(1.6).

(i) The random variable R is the sum of three independent values of X. Find $P(R < 4)$. [3]

(ii) The random variable S is the sum of n independent values of X. It is given that

$$P(S = 4) = \frac{16}{3} \times P(S = 2).$$

Find n. [4]

(iii) The random variable T is the sum of 40 independent values of X. Find $P(T > 75)$. [4]

Q10.

The probability that a new car of a certain type has faulty brakes is 0.008. A random sample of 520 new cars of this type is chosen, and the number, X, having faulty brakes is noted.

(i) Describe fully the distribution of X and describe also a suitable approximating distribution. Justify this approximating distribution. [4]

(ii) Use your approximating distribution to find

(a) $P(X > 3)$, [2]

(b) the smallest value of n such that $P(X = n) > P(X = n + 1)$. [3]
