Poisson Distribution 2

Q1.

The independent random variables X and Y have the distributions $\mathrm{Po}(2)$ and $\mathrm{Po}(3)$ respectively.

(i) Given that $X + Y = 5$, find the probability that $X = 1$ and $Y = 4$. [4]

(ii) Given that $\mathrm{P}(X = r) = \frac{2}{3}\mathrm{P}(X = 0)$, show that $3 \times 2^{r-1} = r!$ and verify that $r = 4$ satisfies this equation. [2]

Q2.

The number of radioactive particles emitted per 150-minute period by some material has a Poisson distribution with mean 0.7.

(i) Find the probability that at most 2 particles will be emitted during a randomly chosen 10-hour period. [3]

(ii) Find, in minutes, the longest time period for which the probability that no particles are emitted is at least 0.99. [5]

Q3.

Goals scored by Femchester United occur at random with a constant average of 1.2 goals per match. Goals scored against Femchester United occur independently and at random with a constant average of 0.9 goals per match.

(i) Find the probability that in a randomly chosen match involving Femchester,

(a) a total of 3 goals are scored, [2]

(b) a total of 3 goals are scored and Femchester wins. [3]

The manager promises the Femchester players a bonus if they score at least 35 goals in the next 25 matches.

(ii) Find the probability that the players receive the bonus. [4]

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Q4.

The proportion of people who have a particular gene, on average, is 1 in 1000. A random sample of 3500 people in a certain country is chosen and the number of people, X, having the gene is found.

(i) State the distribution of X and state also an appropriate approximating distribution. Give the values of any parameters in each case. Justify your choice of the approximating distribution. [3]

(ii) Use the approximating distribution to find $\mathrm{P}(X \leqslant 3)$. [2]

Q5.

(i) The random variable W has the distribution $\mathrm{Po}(1.5)$. Find the probability that the sum of 3 independent values of W is greater than 2. [3]

(ii) The random variable X has the distribution $\mathrm{Po}(\lambda)$. Given that $\mathrm{P}(X = 0) = 0.523$, find the value of λ correct to 3 significant figures. [2]

(iii) The random variable Y has the distribution $\mathrm{Po}(\mu)$, where $\mu \neq 0$. Given that

$$\mathrm{P}(Y = 3) = 24 \times \mathrm{P}(Y = 1),$$

find μ. [3]

Q6.

On average 1 in 25 000 people have a rare blood condition. Use a suitable approximating distribution to find the probability that fewer than 2 people in a random sample of 100 000 have the condition. [3]

Q7.

In a certain lottery, 10 500 tickets have been sold altogether and each ticket has a probability of 0.0002 of winning a prize. The random variable X denotes the number of prize-winning tickets that have been sold.

(i) State, with a justification, an approximating distribution for X. [3]

(ii) Use your approximating distribution to find $\mathrm{P}(X < 4)$. [3]

(iii) Use your approximating distribution to find the conditional probability that $X < 4$, given that $X \geqslant 1$. [4]

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Q8.

The battery in Sue's phone runs out at random moments. Over a long period, she has found that the battery runs out, on average, 3.3 times in a 30-day period.

(i) Find the probability that the battery runs out fewer than 3 times in a 25-day period. [3]

(ii) **(a)** Use an approximating distribution to find the probability that the battery runs out more than 50 times in a year (365 days). [4]

(b) Justify the approximating distribution used in part **(ii)(a)**. [1]

(iii) Independently of her phone battery, Sue's computer battery also runs out at random moments. On average, it runs out twice in a 15-day period. Find the probability that the total number of times that her phone battery and her computer battery run out in a 10-day period is at least 4. [3]

Q9.

At a certain shop the demand for hair dryers has a Poisson distribution with mean 3.4 per week.

(i) Find the probability that, in a randomly chosen two-week period, the demand is for exactly 5 hair dryers. [3]

(ii) At the beginning of a week the shop has a certain number of hair dryers for sale. Find the probability that the shop has enough hair dryers to satisfy the demand for the week if

(a) they have 4 hair dryers in the shop, [2]

(b) they have 5 hair dryers in the shop. [2]

(iii) Find the smallest number of hair dryers that the shop needs to have at the beginning of a week so that the probability of being able to satisfy the demand that week is at least 0.9. [3]
