Poisson Distribution 2 MS

Q1.

4	**(i)**	$e^{-2} \times 2\ (\times)\ e^{-3} \times \frac{3^4}{4!}$	M1	Correct exp'n for P(1) with λ=2 OR P(4) with λ=3
		$e^{-5} \times \frac{5^4}{5!}$	B1	Correct exp'n
		$\div$	M1	dep M1B1
		$\frac{162}{625}$ or 0.259 (3 sf)	A1 4	
	(ii)	$(e^{-2} \times \frac{2^r}{r!} = \frac{2}{3}e^{-2} \Rightarrow)$		
		$3 \times 2^r = 2 \times r!$ OR $2^{r-1} = \frac{1}{3} \times r!$	B1	Legitimately shown
		$(\Rightarrow 3 \times 2^{r-1} = r!)$ $3 \times 2^3 = 24$ OR $3! = 24$ seen	B1 2	Legitimately shown on either equation
			[Total: 6]	

Q2.

4 (i)	$\lambda = 2.8$	B1	seen
	$e^{-2.8}(1 + 2.8 + \frac{2.8^2}{2})$	M1	any λ allow one end error
	$= 0.469$ (3 s.f.) or 0.47(0)	A1 [3]	As final answer
(ii)	$e^{-0.7n} \geqslant 0.99$ or $e^{-\lambda} \geqslant 0.99$	M1	Allow '=' throughout
	$-0.7n \geqslant \ln 0.99$ or $-\lambda \geqslant \ln 0.99$	M1	Attempt ln both sides
	$n \leqslant 0.01436$ or $\lambda \leqslant 0.01005$	A1	Can be implied. Accept 3 s.f.
	'0.01436' × 150 or '0.01005' × 150 ÷ 0.7	M1	Note $e^{-(0.7/150)n} \geqslant 0.99$ scores 1st and 3rd M1
	Max period is 2.15 mins (3 sf)	A1 [5]	T & I leading to ans 2.2 mins, SC: B2

Q3.

4	**(i) (a)**	$e^{-2.1} \times \frac{2.1^3}{3!}$ alone	**M1**		Allow any λ. Allow sum of 3 or 4 rel products, e.g. P (3, 0)
		$= 0.189$	**A1**	**2**	
	(b)	$e^{-1.2} \times \frac{1.2^3}{3!} \times e^{-0.9}$	**M1**		P (Fem = 3) × P (Opp = 0) or P (Fem = 2) × P (Opp = 1)
		$+ e^{-1.2} \times \frac{1.2^2}{2!} \times e^{-0.9} \times 0.9$	**M1**		P (3, 0) + P (2, 1)
		$= 0.115$	**A1**	**3**	As final answer
	(ii)	N (30, 30)	**B1**		seen or implied
		$\frac{34.5-30}{\sqrt{30}}$ (= 0.8216)	**M1**		standardising with their N (λ, λ)
		$1 - \Phi$("0.822")	**M1**		Allow with no or incorrect cc or no √
		= 0.206 (3sf)	**A1**	**4**	Area consistent with their working
Total					

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Q4.

4	(i)	B(3500, 0.001) Poisson with mean = 3.5 $n > 50$ and $np < 5$	B1 B1 B1 [3]	or Po(3.5) Both. Or $n > 50$ and $\lambda < 5$ or $3.5 < 5$
	(ii)	$e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2} + \frac{3.5^3}{3!})$ = 0.537 (3 dp)	M1 A1 [2]	Allow any λ
			[Total: 5)	

Q5.

4	(i)	$\lambda = 4.5$ $1 - e^{-4.5}\left(1 + 4.5 + \frac{4.5^2}{2}\right)$ = 0.826 (3 s.f.)	B1 M1 A1 [3]	seen any λ. Allow one end error
	(ii)	$e^{-\lambda} = 0.523$ $(-\lambda = \ln 0.523)$ $\lambda = 0.648$ (3 s.f.)	B1 B1 [2]	
	(iii)	$e^{-\mu} \times \frac{\mu^3}{3!} = 24 \times e^{-\mu} \times \mu$ $\frac{\mu^2}{6} = 24$ $\mu = 12$	B1 M1 A1 [3]	 For a simplified expression in μ^2 with $e^{-\mu}$ and μ cancelled and no factorials.

Q6.

1	$e^{-4}(1 + 4)$ = 0.0916 (3 s.f.)	M1 M1 A1 [3]	M1 for P(0 or 1) using Poisson, any λ Expression of correct form correct λ (allow 1 end error) SR Use of Bin(100000, 1/25000) scores M1 for P(0,1) allow one end error. A1 0.0916

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Q7.

7	(i)	Poisson (Actually binomial with) $n > 50$ and np (or λ) (= 2.1) which is < 5	B1 B1 B1 3	 Allow without "binomial" Accept n large Accept p small (p < 0.1)
	(ii)	$\lambda = 2.1$ $e^{-2.1}\left(1+2.1+\frac{2.1^2}{2}+\frac{2.1^3}{3!}\right)$ = 0.839 (3 sf)	B1 M1 A1 3	 Attempt P(0,1,2,3) any λ allow 1 end error SR_1 Ft Normal N(2.1,2.1) B1 standardising M1 0.833 A1 SR_2 Ft Binomial B(10500,0.0002) B1 calculating binomial prob P(0,1,2,3) M1 = 0.8386 A1
	(iii)	$P(X \geqslant 1) = 1 - e^{-2.1}$ (= 0. 87754) $P(X = 1,2,3) = e^{-2.1}\left(2.1+\frac{2.1^2}{2}+\frac{2.1^3}{3!}\right)$ (= 0.71619) $\frac{P(X=1,2,3)}{P(X>1)}$ $\left(=\frac{0.71619}{0.87754}\right)$ = 0.816 (3 sf)	M1 M1 M1 A1 4	Any λ Or '0.839' – $e^{-2.1}$ Any λ Allow any attempted $\frac{P(X=1,2,3)}{P(X>1)}$ Any λ SR_1 Ft Normal P(>0.5)=0.86523 M1 P(1,2,3)=0.698 M1 0.698/0.86523 = 0.807 M1A1 SR_2 FT Binomial M1 M1 M1 A1
		Total	**10**	

Q8.

6	(i)	$\lambda = 3.3 \times \frac{25}{30} = 2.75$ $e^{-2.75}(1 + 2.75 + \frac{2.75^2}{2})$ = 0.481 (3 sf)	**B1** **M1** **A1**	 [3]	 Allow any λ Allow one end error As final answer. Accept 0.482
	(ii) (a)	λ (= $3.3 \times \frac{365}{30}$) = 40.15 ($X \sim Po(40.15) \Rightarrow X \sim N(40.15, 40.15)$) $\frac{50.5-"40.15"}{\sqrt{"40.15"}}$ (= 1.633) 1 – Φ("1.633") = 0.0513 (3 sf)	**B1** **M1** **M1** **A1**	 [4]	Accept 40.1 or 40.2 Allow with incorrect or no cc OR no √ sign For correct area consistent with their working Accept 0.0512
	(b)	$\lambda > 15$	**B1**	[1]	or similar
	(iii)	$\lambda = \frac{73}{30}$ oe or 1.1+1.33= 2.43 (3 sf) $1 - e^{-2.43}(1 + 2.43 + \frac{2.43^2}{2} + \frac{2.43^3}{3!})$ = 0.228 (3 sf)	**B1** **M1** **A1**	 [3]	 Allow any λ. Allow one end error

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Q9.

6 (i)	$\lambda = 6.8$ $e^{-6.8} \times \frac{6.8^5}{5!}$ $= 0.135$ (3 sf)	**B1** **M1** **A1**	 [3]	 any λ
(ii) (a)	$e^{-3.4}(1 + 3.4 + \frac{3.4^2}{2} + \frac{3.4^3}{3!} + \frac{3.4^4}{4!})$ $= 0.744$ (3 sf)	**M1** **A1**	 [2]	any λ, allow one end-error
(b)	'0.744' $+ e^{-3.4} \times \frac{3.4^5}{5!}$ $= 0.87(0)$ (3 sf) or 0.871	**M1** **A1**	 [2]	or complete method, any λ, allow one end-error
(iii)	$P(X \leqslant 6) =$ '0.870' $+ e^{-3.4} \times \frac{3.4^6}{6!}$ $= 0.94$ Need 6 hair driers	**M1** **A1** **A1**	 [3]	or complete method, any λ fully correct un-simplified expression or better dep M1A1 with numerical justification (0.94 or better)