Polar Coordinates 1 - MS

Q1. **(i)** $\theta = 2, r = 4$ B1B1
Ignore extra values
Accept as written in MS – co-ords not required

(ii) Graphs: correct location, orientation and concavity required B1B1
Separate diagrams 1/2, B1 Shapes correct
B1 Intersection correct

(iii) $A_1 = (1/2)\ \int_0^2 (\theta + 2)^2\, d\theta$ (LNR) M1

$= \ldots = 28/3$

$A_2 = (1/2)\ \int_0^2 \theta^4\, d\theta = \ldots = 16/5$ (LR) A1

M1 for 1 correct integral representation plus A1 if both correct

Area $= A_1 - A_2 = 92/15$ (6.13) A1

S.C. –92/15 M1 A0 A1

Alternative layout: $\frac{1}{2}\int_0^2 (\theta + 2)^2 - \theta^4\, d\theta$ M1 (LNR)

$= \frac{1}{2}\left[\frac{\theta^3}{3} + 2\theta^2 + 4\theta - \frac{\theta^5}{5}\right]_0^2$ A1 (LR)

$= 92/15$ A1

Q2. Approximately correct curve passing through the pole, O, and the point $A(\pi^2/4,0)$. B1

Negative gradient at A B1

Correct form at O. B1

$\text{Area} = (1/2)\int_0^{\pi/2} (\pi/2 - \theta)^4\, d\theta$ M1

$= -(1/10)\left[(\pi/2 - \theta)^5\right]_0^{\pi/2}$ A1

$= \pi^5/320$ A1

Q3. Sketch with correct shape, location and orientation B1

Shows tangency to the initial line at the pole B1

Ignore extra in diagram

Draws, in the same diagram, a straight line passing through the origin and with positive gradient (distinct half-line and not a construction line) B1

$(1/2)\int_0^{\pi/2}\theta^2 d\theta = \pi^3/48$ M1A1

$(1/2)\int_0^{\alpha}\theta^2 d\theta = \alpha^3/6$ A1

$\alpha^3/6 = \pi^3/96 \Rightarrow \alpha = \pi.2^{-4/3}$ (aef) A1

or for previous 4 marks:

$(1/2)\int_0^{\alpha}\theta^2 d\theta = (1/2)\int_{\alpha}^{\pi/2}\theta^2 d\theta$ M1

$\alpha^3/6 = (1/6)[(\pi/2)^3 - \alpha^3]$ A1A1

$\Rightarrow \alpha^3 = \pi^3/16 \Rightarrow \alpha = \pi(16)^{-1/3}$, or equivalent A1

Q4. **(i)** Sketch of C:

Approximately correct shape and location for $0 \le \theta < 2\pi$ B1
Shows initial line to be tangential to C at the pole B1
Asymptotic approach to circle $r = a$ B1
[3]

(ii) $A = (a^2/2)\int_{\ln 2}^{\ln 4}(1 - 2e^{-\theta} + e^{-2\theta})d\theta$ M1A1
$= (a^2/2)[\theta + 2e^{-\theta} - (1/2)e^{-2\theta}]_{\ln 2}^{\ln 4}$ A1
$= \ldots = (a^2/2)(\ln 2 - 13/32)$ (AG) A1
[4]

Q5.

5	Right-hand loop Left-hand loop Deduct 1 mark for extra loops ($r < 0$).	Position and through pole and (2, 0). Position and through pole and (2, π).	B1B1 B1B1	4	
	Uses $A = \frac{1}{2}\int r^2 \, d\theta$	$A = \frac{1}{2}\int 4\cos^2\theta \, d\theta \, d\theta$	M1		
	Uses double angle formula.	$= \int (\cos 4\theta + 1) \, d\theta$ (LNR)	M1		
	Integrates.	$= [\sin 4\theta + \theta]$ (LNR)	A1		
	Inserts *any appropriate* limits which legitimately give the result.	e.g. $[\sin 4\theta + \theta]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi}{2}$	A1	4	**[8]**

Q6.

6	Sketches each curve on same diagram.	Sketch of C_1 (relevant part only required). Sketch of C_2 (generous on tangency features).	B1 B2	3	
	States the value of β.	$\beta = \frac{\pi}{6}$.	B1	1	
	Adds $\frac{1}{12}$ of area of circle to sector of C_2 from $\theta = \frac{\pi}{6}$ to $\theta = \frac{\pi}{2}$.	$\frac{1}{12}\pi a^2 + \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 4a^2 \cos^2 2\theta d\theta$	B1M1		
	Uses double angle formula and integrates.	$= \frac{1}{12}\pi a^2 + a^2 \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\cos 4\theta + 1) d\theta$			
		$= \frac{1}{12}\pi a^2 + a^2 \left[\frac{\sin 4\theta}{4} + \theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$	M1		
	Obtains printed result.	$= a^2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right)$ (AG)	A1	4	**[8]**

Q7.

	C:	Closed loop through (5,0) and $(1,\pi)$ Correct shape near $(1,\pi)$	B1 B1	3	
	Straight line	Perpendicular to initial line , through (2,0)	B1		
		$\Rightarrow (3+2\cos\theta)\cos\theta = 2$			
	Forms quadratic equation in usual form.	$\Rightarrow 2\cos^2\theta + 3\cos\theta - 2 = 0$ (aef)	M1		
		$\Rightarrow (2\cos\theta - 1)(\cos\theta + 2) = 0$			
	Solves quadratic equation.	$\Rightarrow \cos\theta = 0.5$ (since $\cos\theta > 0$)	A1		
	Writes down points of intersection.	Intersections at $\left(4,\frac{\pi}{3}\right)$ and $\left(4,-\frac{\pi}{3}\right)$.	A1A1	4	
		Calling points of intersection A and B and the pole O. Required area is two congruent sectors between l and C plus triangle OAB.			
	Finds required area.	Two sectors $= 2\times\frac{1}{2}\int_{\frac{\pi}{3}}^{\pi}(9+12\cos\theta+4\cos^2\theta)d\theta$	M1		
		$= \int_{\frac{\pi}{3}}^{\pi}(11+12\cos\theta+2\cos 2\theta)d\theta$	A1		
		$= \left[11\theta+12\sin\theta+\sin 2\theta\right]_{\frac{\pi}{3}}^{\pi}$	M1		
		$= \frac{22\pi}{3} - \frac{13\sqrt{3}}{2} = (11.78)$	A1		
		Triangle $= 2\sqrt{3}\times 2 = 4\sqrt{3} = (6.928)$	B1		
		Total Area $= \frac{22\pi}{3} - \frac{5\sqrt{3}}{2} = (18.708 = 18.7\ (3sf))$	A1	6	**[13]**

Q8.

8	Sketches graph.	Arc above initial line. Arc below initial line.	B1 B1	2	
	Uses $\frac{1}{2}\int r^2 d\theta$	$\frac{1}{2}\int(1+\sin\theta)^2\,\mathrm{d}\theta = \frac{1}{2}\int(1+2\sin\theta+\sin^2\theta)\,\mathrm{d}\theta$	M1		
	Uses double angle formula.	$= \frac{1}{2}\int\left(\frac{3}{2}+2\sin\theta-\frac{1}{2}\cos 2\theta\right)\mathrm{d}\theta$	M1		
	Integrates.	$= \frac{1}{2}\left[\frac{3\theta}{2}-2\cos\theta-\frac{1}{4}\sin 2\theta\right]+c$	M1A1		
	Inserts limits.	$A_1 = \left[\frac{1}{2}\left(\frac{3\theta}{2}-2\cos\theta-\frac{1}{4}\sin 2\theta\right)\right]_0^{\frac{\pi}{2}} = \frac{3\pi}{8}+1$	M1A1		
		$A_2 = \left[\frac{1}{2}\left(\frac{3\theta}{2}-2\cos\theta-\frac{1}{4}\sin 2\theta\right)\right]_{-\frac{\pi}{2}}^{0} = \frac{3\pi}{8}-1$	A1		
		$n = \left(\frac{3\pi}{8}+1\right)\div\left(\frac{3\pi}{8}-1\right) = 12.2$ (1d.p.)	A1	8	**[10]**

Q9.

	Uses $x^2+y^2=r^2$, $x=r\cos\theta$ and $y=r\sin\theta$.	$(x^2+y^2)^2 = a^2(x^2-y^2)$ $\Rightarrow r^2 = a^2\left(\frac{x^2}{r^2}-\frac{y^2}{r^2}\right)$ $= a^2(\cos^2\theta-\sin^2\theta) = a^2\cos 2\theta$ (AG)	M1 A1	 2	
	One mark for each loop, or half of whole curve.	Sketches C.	B2,1,0	2	
	Uses sector area formula.	Area $= \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} a^2\cos 2\theta\,\mathrm{d}\theta\left(=\int_0^{\frac{\pi}{4}} a^2\cos 2\theta\,\mathrm{d}\theta\right)$	M1		
	S.C. Omission of ½ factor, but correct integration gets B1.	$= a^2\left[\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}} = \frac{a^2}{2}$	A1A1	3	
	Differentiates. Puts $y'=0$. Obtains coordinates.	$2(x^2+y^2)(2x+2yy') = a^2(2x-2yy')$ $y'=0 \Rightarrow 2x(x^2+y^2)=a^2x$ $\Rightarrow 2r^2=a^2 \Rightarrow r=\frac{a}{\sqrt{2}}$ $(r\ge 0)$ $\Rightarrow \cos 2\theta = \frac{1}{2}$ $\Rightarrow \theta = \pm\frac{\pi}{6},\ \pm\frac{5\pi}{6}$ i.e. $\left(\frac{a}{\sqrt{2}},\pm\frac{\pi}{6}\right)$ and $\left(\frac{a}{\sqrt{2}},\pm\frac{5\pi}{6}\right)$	B1B1 M1 A1 M1 A1A1	 7	**[14]**