# Polar Coordinates 2

Q1. Use the identity $2\sin P\cos Q \equiv \sin(P+Q)+\sin(P-Q)$ to show that

$$2\sin\theta\cos(\theta-\tfrac{1}{4}\pi) \equiv \cos(2\theta-\tfrac{3}{4}\pi)+\frac{1}{\sqrt{2}}.$$ [3]

A curve has polar equation $r = 2\sin\theta\cos(\theta-\frac{1}{4}\pi)$, for $0 \leqslant \theta \leqslant \frac{3}{4}\pi$. Sketch the curve and state the polar equation of its line of symmetry, justifying your answer. [3]

Show that the area of the region enclosed by the curve is $\frac{3}{8}(\pi+1)$. [6]

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Q2. The curve $C$ has polar equation $r = 2e^{\theta}$, for $\frac{1}{6}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$. Find

**(i)** the area of the region bounded by the half-lines $\theta = \frac{1}{6}\pi$, $\theta = \frac{1}{2}\pi$ and $C$, [2]

**(ii)** the length of $C$. [3]

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Q3. The curve $C$ has polar equation $r = 2\sin\theta(1-\cos\theta)$, for $0 \leqslant \theta \leqslant \pi$. Find $\dfrac{dr}{d\theta}$ and hence find the polar coordinates of the point of $C$ that is furthest from the pole. [5]

Sketch $C$. [2]

Find the exact area of the sector from $\theta = 0$ to $\theta = \frac{1}{4}\pi$. [6]

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Q4. A circle has polar equation $r = a$, for $0 \leqslant \theta < 2\pi$, and a cardioid has polar equation $r = a(1-\cos\theta)$, for $0 \leqslant \theta < 2\pi$, where $a$ is a positive constant. Draw sketches of the circle and the cardioid on the same diagram. [3]

Write down the polar coordinates of the points of intersection of the circle and the cardioid. [2]

Show that the area of the region that is both inside the circle and inside the cardioid is

$$(\tfrac{5}{4}\pi-2)a^2.$$ [6]

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Q5. The curves $C_1$ and $C_2$ have polar equations

$$C_1:\quad r = \frac{1}{\sqrt{2}}, \quad \text{for } 0 \leqslant \theta < 2\pi,$$

$$C_2:\quad r = \sqrt{(\sin\tfrac{1}{2}\theta)}, \quad \text{for } 0 \leqslant \theta \leqslant \pi.$$

Find the polar coordinates of the point of intersection of $C_1$ and $C_2$. [2]

Sketch $C_1$ and $C_2$ on the same diagram. [3]

Find the exact value of the area of the region enclosed by $C_1$, $C_2$ and the half-line $\theta = 0$. [4]

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Q6. A curve $C$ has polar equation $r^2 = 8 \operatorname{cosec} 2\theta$ for $0 < \theta < \frac{1}{2}\pi$. Find a cartesian equation of $C$. [3]

Sketch $C$. [2]

Determine the exact area of the sector bounded by the arc of $C$ between $\theta = \frac{1}{6}\pi$ and $\theta = \frac{1}{3}\pi$, the half-line $\theta = \frac{1}{6}\pi$ and the half-line $\theta = \frac{1}{3}\pi$. [3]

[It is given that $\int \operatorname{cosec} x \, dx = \ln\left|\tan \frac{1}{2}x\right| + c$.]

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Q7. The curves $C_1$ and $C_2$ have polar equations, for $0 \leqslant \theta \leqslant \pi$, as follows:

$$C_1 : r = a,$$
$$C_2 : r = 2a|\cos\theta|,$$

where $a$ is a positive constant. The curves intersect at the points $P_1$ and $P_2$.

**(i)** Find the polar coordinates of $P_1$ and $P_2$. [2]

**(ii)** In a single diagram, sketch $C_1$, $C_2$ and their line of symmetry. [3]

**(iii)** The region $R$ enclosed by $C_1$ and $C_2$ is bounded by the arcs $OP_1$, $P_1P_2$ and $P_2O$, where $O$ is the pole. Find the area of $R$, giving your answer in exact form. [5]

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Q8. The curve $C$ has polar equation

$$r = 5\sqrt{(\cot\theta)},$$

where $0.01 \leqslant \theta \leqslant \frac{1}{2}\pi$.

**(i)** Find the area of the finite region bounded by $C$ and the line $\theta = 0.01$, showing full working. Give your answer correct to 1 decimal place. [3]

Let $P$ be the point on $C$ where $\theta = 0.01$.

**(ii)** Find the distance of $P$ from the initial line, giving your answer correct to 1 decimal place. [2]

**(iii)** Find the maximum distance of $C$ from the initial line. [3]

**(iv)** Sketch $C$. [2]

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Q9. The curve $C$ has polar equation $r = a\cos 3\theta$, for $-\frac{1}{6}\pi \leqslant \theta \leqslant \frac{1}{6}\pi$, where $a$ is a positive constant.

**(i)** Sketch $C$. [2]

**(ii)** Find the area of the region enclosed by $C$, showing full working. [3]

**(iii)** Using the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$, find a cartesian equation of $C$. [3]

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