# Rational Functions and Graphs 1

The curve $C$ has equation

$$y = \frac{(x-2)(x-a)}{(x-1)(x-3)},$$

where $a$ is a constant not equal to 1, 2 or 3.

**(i)** Write down the equations of the asymptotes of $C$. [2]

**(ii)** Show that $C$ meets the asymptote parallel to the $x$-axis at the point where $x = \frac{2a-3}{a-2}$. [2]

**(iii)** Show that the $x$-coordinates of any stationary points on $C$ satisfy

$$(a-2)x^2 + (6-4a)x + (5a-6) = 0,$$

and hence find the set of values of $a$ for which $C$ has stationary points. [6]

**(iv)** Sketch the graph of $C$ for

**(a)** $a > 3$,

**(b)** $2 < a < 3$.

[4]

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**3** The curve $C$ has equation

$$y = \frac{x^2 - 5x + 4}{x + 1}.$$

**(i)** Obtain the coordinates of the points of intersection of $C$ with the axes. [2]

**(ii)** Obtain the equation of each of the asymptotes of $C$. [3]

**(iii)** Draw a sketch of $C$. [3]

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**6** The curve $C$ has equation

$$y = \frac{x^2 - 3x - 7}{x + 1}.$$

**(i)** Obtain the equations of the asymptotes of $C$. [3]

**(ii)** Show that $\frac{dy}{dx} > 1$ at all points of $C$. [2]

**(iii)** Draw a sketch of $C$. [3]

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The curve $C$ has equation

$$y = \frac{x(x+1)}{(x-1)^2}.$$

**(i)** Obtain the equations of the asymptotes of $C$. [3]

**(ii)** Show that there is exactly one point of intersection of $C$ with the asymptotes and find its coordinates. [2]

**(iii)** Find $\frac{dy}{dx}$ and hence

**(a)** find the coordinates of any stationary points of $C$,

**(b)** state the set of values of $x$ for which the gradient of $C$ is negative.

[6]

**(iv)** Draw a sketch of $C$. [3]

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**9** The curve $C$ with equation

$$y = \frac{ax^2 + bx + c}{x - 1},$$

where $a$, $b$ and $c$ are constants, has two asymptotes. It is given that $y = 2x - 5$ is one of these asymptotes.

**(i)** State the equation of the other asymptote. [1]

**(ii)** Find the value of $a$ and show that $b = -7$. [3]

**(iii)** Given also that $C$ has a turning point when $x = 2$, find the value of $c$. [3]

**(iv)** Find the set of values of $k$ for which the line $y = k$ does not intersect $C$. [4]

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The curve $C$ has equation $y = \frac{x^2 + px + 1}{x - 2}$, where $p$ is a constant. Given that $C$ has two asymptotes, find the equation of each asymptote. [3]

Find the set of values of $p$ for which $C$ has two distinct turning points. [5]

Sketch $C$ in the case $p = -1$. Your sketch should indicate the coordinates of any intersections with the axes, but need not show the coordinates of any turning points. [3]

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