Rational Functions and Graphs 1 - MS

Q1.	**(i)**	$x = 1, x = 3$ (both)	B1
		$y = 1$	B1
	(ii)	Solves $(x-2)(x-a)/(x-1)(x-3) = 1$ to obtain $x = \xi$ where $\xi = (2a-3)/(a-2)$	M1A1
	(iii)	$y_1 = 0 \Rightarrow (x-2)(x-a)(2x-4) = (x-1)(x-3)(2x-2-a)$	M1
		$\Rightarrow (-4-4-2a)x^2 + (4a+8+4a)x - 8a = (-8-2-a)x^2 + (6+8+4a)x - 6 - 3a$	A1
		$\Rightarrow (a-2)x^2 + (6-4a)x + (5a-6) = 0$ (AG)	A1
		$(6-4a)^2 \geqslant 4(a-2)(5a-6)$	M1
		$\Rightarrow a^2 - 4a + 3 \leqslant 0 \Rightarrow (a-1)(a-3) \leqslant 0$	M1
		$\Rightarrow 1 < a < 3$ ($a \neq 2$ given)	A1
	(iv) (a)	Axes and asymptotes	B1
		Branches (all)	B1
	(b)	Middle branch with maximum value in the range $0 < y < 1$	B1
		Outside branches with correctly placed minimum point	B1

Q2.	**(i)**	(1,0), (4,0)	B1
		(0,4)	B1
	(ii)	One asymptote is $x = -1$	B1
		$y = x - 6 + 10/(x+1)$	M1
		Other asymptote: $y = x - 6$	A1
	(iii)	Sketch:	
		Axes and asymptotes	B1
		Upper branch:	
		Correct location and orientation	B1
		Lower branch correctly located and orientated	B1

Q3.	**(i)**	One asymptote is $x = -1$	B1
		$y = x - 4 - 3/(x+1)$	M1
		Require $y = x +$ non-zero constant.	
		Other asymptote is $y = x - 4$	A1
			[3]
		Alternatively for last two marks:	
		OR $x + k \approx (x^2 - 3x - 7)/(x+1)$ for large $x \Rightarrow x^2 + (k+1)x + k \approx x^2 - 3x - 7$ for large x	M1
		$\Rightarrow k + 1 = -3 \Rightarrow k = -4 \Rightarrow$ other asymptote is $y = x - 4$	A1

Q4. **(i)** $x = 1$ B1

$y = 1 + O(1/x)$ as $|x| \to \infty$ M1

Second asymptote is $y = 1$ A1

[3]

(ii) $x(x + 1) / (x - 1)^2 = 1 \Rightarrow x = 1/3, y = 1$ M1A1

[2]

(iii) (a) $dy / dx = 0 \Rightarrow [(2x + 1)(x - 1)^2 - 2x(x - 1)(x + 1)] / (x - 1)^4 = 0$ M1

$\Rightarrow x = 1/3, y = -1/8$ M1A1

(b) $dy / dx = -(3x + 1) / (x - 1)^3$ M1

$\{x : x < -1/3\} \cup \{x : x > 1\}$ A1√A1

ft.

[6]

(iv) Sketch:

Left-hand branch with approximately correct shape and location and passing through the origin and (–1, 0). B1

Intersection with $y = 1$ and location of minimum point consistent with results of **(ii)** and **(iii)** (cwo) B1

Right-hand branch with approximately correct forms at infinity B1

[3]

Q5.

(i)	States vertical asymptote.	$x = 1$	B1	1	
(ii)	States the value of a. Divides. Compares coefficients to obtain b.	$a = 2$ $y = ax + a + b + \frac{a+b+c}{(x-1)}$ $2 + b = -5 \Rightarrow b = -7$ (AG) Or $y = 2x - 5 + \frac{a}{x-1}$ $= \frac{2x^2 - 7x + 5 + a}{x-1}$ Equate coefficients to obtain $a = 2,\ b = -7$	B1 M1 A1 (M1) (B1A1)	3	

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(iii)	Differentiates and uses given value of x to obtain c.	$y' = 2 - \frac{(c-5)}{(x-1)^2} = 0$	M1A1		
		When $x = 2$ then $c = 7$	A1	3	
(iv)	Forms quadratic in x.	Let $y = \frac{2x^2 - 7x + 7}{(x-1)} = k$			
		$\Rightarrow 2x^2 - (7+k)x + 7 + k = 0$	B1		
	Uses discriminant.	No real roots $\Rightarrow (7+k)^2 - 8(7+k) < 0$	M1		
		$\Rightarrow k^2 + 6k - 7 < 0$	A1		
		$\Rightarrow (k+7)(k-1) < 0$			
	Obtains required result.	$\Rightarrow -7 < k < 1$	A1	4	**[11]**

Q6.

	Vertical asymptote.	$x = 2$	B1	3	
	Divides by $(x-2)$	$y = x + p + 2 + \frac{2p+5}{x-2}$	M1		
	Oblique asymptote.	$y = x + p + 2$	A1	3	
	Differentiates.	$\frac{dy}{dx} = \frac{x^2 - 4x + 4 - 2p - 5}{(x-2)^2}$	M1A1		
		$y' = 0 \Rightarrow x^2 - 4x - (2p+1) = 0$	M1		
		$B^2 - 4AC > 0 \Rightarrow 16 + 4(2p+1) > 0$	M1		
		$\Rightarrow p > -\frac{5}{2}$	A1	5	
	Sketches graph. Working to show either $x^2 - x + 1 = 0$ has no real roots, or maximum value.	Axes and (0,–0.5) marked.. Upper Branch with minimum. Lower with maximum below x-axis. (Deduct at most 1 for poor forms at infinity.)	B1 B1 B1	3	**[11]**