Rational Functions and Graphs 2

Q1. A curve C has equation

$$y = \frac{5(x^2 - x - 2)}{x^2 + 5x + 10}.$$

Find the coordinates of the points of intersection of C with the axes. [2]

Show that, for all real values of x, $-1 \leqslant y \leqslant 15$. [4]

Sketch C, stating the coordinates of any turning points and the equation of the horizontal asymptote. [7]

Q2. The curve C has equation

$$y = \frac{2x^2 + 2x + 3}{x^2 + 2}.$$

Show that, for all x, $1 \leqslant y \leqslant \frac{5}{2}$. [4]

Find the coordinates of the turning points on C. [3]

Find the equation of the asymptote of C. [2]

Sketch the graph of C, stating the coordinates of any intersections with the y-axis and the asymptote. [2]

Q3. The curve C has equation $y = \frac{x^2}{x - 2}$. Find the equations of the asymptotes of C. [3]

Find the coordinates of the turning points on C. [3]

Draw a sketch of C. [3]

Q4. The curve C has equation

$$y = \frac{x^2 + b}{x + b},$$

where b is a positive constant.

(i) Find the equations of the asymptotes of C. [3]

(ii) Show that C does not intersect the x-axis. [1]

(iii) Justifying your answer, find the number of stationary points on C. [2]

(iv) Sketch C. Your sketch should indicate the coordinates of any points of intersection with the y-axis. You do not need to find the coordinates of any stationary points. [3]

Q5. The curve C has equation

$$y = \frac{x^2 + 7x + 6}{x - 2}.$$

(i) Find the coordinates of the points of intersection of C with the axes. [2]

(ii) Find the equation of each of the asymptotes of C. [3]

(iii) Sketch C. [3]

Q6. The curve C has equation

$$y = \frac{x^2 + ax - 1}{x + 1},$$

where a is constant and $a > 1$.

(i) Find the equations of the asymptotes of C. [3]

(ii) Show that C intersects the x-axis twice. [1]

(iii) Justifying your answer, find the number of stationary points on C. [2]

(iv) Sketch C, stating the coordinates of its point of intersection with the y-axis. [3]

Q7. The curve C has equation

$$y = \frac{5x^2 + 5x + 1}{x^2 + x + 1}.$$

(i) Find the equation of the asymptote of C. [2]

(ii) Show that, for all real values of x, $-\frac{1}{3} \leqslant y < 5$. [4]

(iii) Find the coordinates of any stationary points of C. [2]

(iv) Sketch C, stating the coordinates of any intersections with the y-axis. [2]

Q8. The curve C has equation $y = \dfrac{x^2 + x + 9}{x + 1}$.

(a) Find the equations of the asymptotes of C. [3]

(b) Find the coordinates of the stationary points on C. [4]

(c) Sketch C, stating the coordinates of any intersections with the axes. [3]

(d) Sketch the curve with equation $y = \left|\dfrac{x^2 + x + 9}{x + 1}\right|$ and find the set of values of x for which $2|x^2 + x + 9| > 13|x + 1|$. [5]
