Rational Functions and Graphs 2 - MS

Q1.

	Intersections with axes.	(–1,0) , (2,0) (0,–1)	B1 B1	2	
		$yx^2+5xy+10y=5x^2-5x-10$			
	Rearranges as a quadratic equation.	$(y-5)x^2+(5y+5)x+10(y+1)=0$			
	Uses discriminant.	For real x $b^2-4ac\geq 0$			
		$\Rightarrow (5y+5)^2-40(y-5)(y+1)\geq 0\ldots$	M1A1		
	Solves inequality.	$\Rightarrow (y-15)(y+1)\leq 0\Rightarrow -1\leq y\leq 15$ (AG)	M1A1	4	
	Finds turning points.	$y=-1\Rightarrow x=0$ $y=15\Rightarrow x=-4$	M1A1 A1		
		Turning points are (-4,15) and (0,–1)			
		$y=5$.	B1		
	States asymptote.	Axes and asymptote correct	B1		
	Sketches graph.	Graph correct.	B1B1	7	**[13]**

Q2.

	Forms quadratic equation in x.	$yx^2+2y=2x^2+2x+3$ $\Rightarrow (y-2)x^2-2x+(2y-3)=0$	M1 A1		
	Uses discriminant to obtain condition for real roots.	For real x $4-4(y-2)(2y-3)\geq 0$ $\Rightarrow (2y-5)(y-1)\leq 0$ $\Rightarrow 1\leq y\leq \frac{5}{2}$ (AG)	M1 A1	4	
	Differentiates and equates to zero. Solves equation.	$y'=0$ $\Rightarrow (x^2+2)(4x+2)-2x(2x^2+2x+3)=0$ $\Rightarrow (x-2)(x+1)=0\Rightarrow x=-1$ or $x=2$ (Or substitutes $y=1$ and $\frac{5}{2}$ in equation of C.)	M1		
	States coordinates of turning points.	Turning points are (–1, 1) and $\left(2, 2\frac{1}{2}\right)$	A1A1	3	
	Expresses y in an appropriate form. (May alternatively divide numerator and denominator by x^2.	$y=2+\frac{2x-1}{x^2+2}$ As $x\to\pm\infty$ $y\to 2$ $\therefore y=2$	M1 A1	2	
	Finds y-intercept and intersection with $y=2$.	Shows $\left(0,1\frac{1}{2}\right)$ and $\left(\frac{1}{2},2\right)$	B1		
	Completes graph.	Completely correct graph.	B1	2	**[11]**

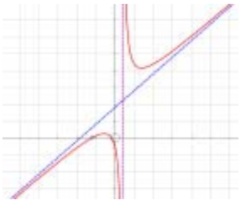
Q3.

	States vertical asymptote.	Vertical asymptote is $x = 2$.	B1		
	Finds oblique asymptote.	$y = x + 2 + \dfrac{4}{x-2}$	M1		
		Oblique asymptote is $y = x + 2$.	A1	3	
	Differentiates and equates to zero.	$y' = 1 - \dfrac{4}{(x-2)^2} = 0 \Rightarrow (x-2)^2 = 4$	M1		
	Finds x coordinates.	$x = 0, 4$.	A1		
	States coordinates of turning points.	Turning points are $(0,0)$ and $(4,8)$	A1	3	
	Deduct at most 1 mark for poor forms at infinity.	Axes and both asymptotes correct. Upper branch correct. Lower branch correct.	B1 B1 B1	3	**[9]**

Q4.

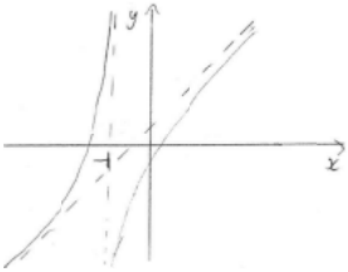
(i)	Vertical asymptote is $x = -b$.	**B1**	
	$x^2 + b = (x+b)(x-b) + b^2 + b$ or $x+b\overline{)x^2 + 0x + b}$ with quotient $x - b$	**M1**	By inspection or long division.
	Thus the oblique asymptote is $y = x - b$	**A1**	
		3	
(ii)	If $y = 0$ then $x^2 + b = 0$ which has no real root.	**B1**	Must refer to b > 0 OE
		1	
(iii)	$\dfrac{dy}{dx} = \dfrac{2x(x+b) - (x^2+b)}{(x+b)^2} = 0 \Rightarrow x^2 + 2bx - b = 0$ Or differentiating $y = x - b + \dfrac{b^2+b}{x+b}$ and setting $\dfrac{dy}{dx} = 0$ gives $1 - \dfrac{b^2+b}{(x+b)^2} = 0$.	**M1**	Find $\dfrac{dy}{dx}$ and set = 0
	$b^2 + b > 0$ Therefore there are two stationary points on C	**A1**	Use discriminant or $(x + b)^2$ to show two stationary points
		2	

Q5.

(i)	$(-6,0),(-1,0)$	**B1**	States points of intersection with x-axis.
	$(0,-3)$	**B1**	States y-intercept
(ii)	One asymptote is $x=2$.	**B1**	
	$y=x+9+\frac{24}{x-2} \Rightarrow$ other asymptote is $y=x+9$.	**M1 A1**	By inspection or long division. A0 if error in division
(iii)		**B1**	Sketches axes and asymptotes, labelled or to scale
		B1	Upper branch correctly located and orientated.
		B1	Lower branch correctly located and orientated. Penalise at most 1 mark for poor forms at infinity
		8	

Q6.

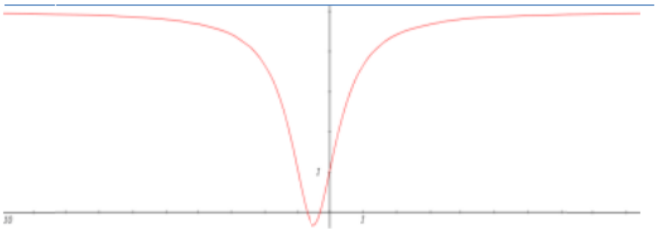
(i)	Vertical asymptote is $x=-1$.	**B1**	
	$x^2+ax-1=(x+1)(x+a-1)-a$ or $x+1\overset{x+a-1}{\overline{\big)x^2+ax-1}}$	**M1**	By inspection or long division.
	Thus the oblique asymptote is $y=x+a-1$	**A1**	
		3	
(ii)	$a^2+4>0$	**B1**	
		1	
(iii)	$\frac{(x+1)(2x+a)-(x^2+ax-1)}{(x+1)^2}=0 \Rightarrow x^2+2x+a+1=0$ Discriminant $=4-4(a+1)<0$	**M1**	
	Therefore there are no stationary points on C.	**A1**	
		2	

(iv)		**B1**	Correct y-intercept and asymptotes drawn.
		B1 B1	Each branch correct
		3	

Q7.

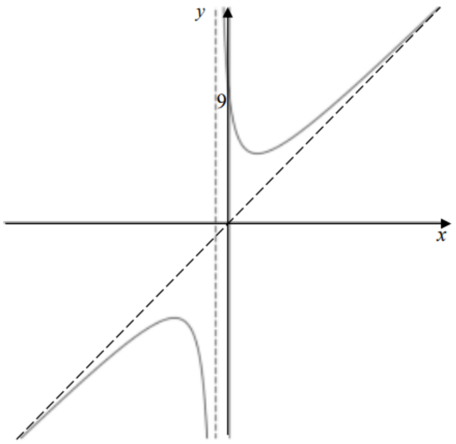
(i)	$y = 5 - \frac{4}{x^2 + x + 1}$	**M1**	Alt method: Finding limit
	As $x \to \pm\infty$, $y \to 5 \therefore y = 5$ CAO	**A1**	
		2	

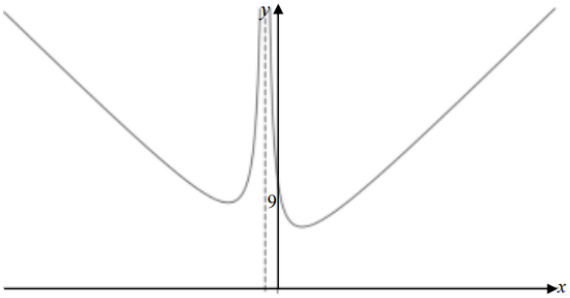
(ii)	$yx^2 + yx + y = 5x^2 + 5x + 1$ $\Rightarrow (y-5)x^2 + (y-5)x + (y-1) = 0$	**B1**	Forms quadratic equation in x.
	For real x, $(y-5)^2 - 4(y-5)(y-1) \geqslant 0$ (condone >)	**M1**	Uses discriminant
	$\Rightarrow (y-5)(3y+1) \leqslant 0$	**M1**	Factorising
	$\Rightarrow -\frac{1}{3} \leqslant y < 5$, because $y = 5$ is an asymptote (www)	**A1**	Explaining strict upper inequality (AG)
		4	
(iii)	$y' = 0 \Rightarrow (x^2 + x + 1)(10x + 5) - (5x^2 + 5x + 1)(2x + 1) = 0$	**M1**	Differentiates and equates to 0.
	$\Rightarrow 4(2x+1) = 0 \Rightarrow x = -\frac{1}{2}, y = -\frac{1}{3}$	**A1**	
		2	

(iv)		**B1FT**	Positive y-intercept at (0,1), FT dep on minimum point from **(iii)**.
		B1	Correct asymptote and completely correct graph.
		2	

Q8.

7(a)	$x = -1$	**B1**	States vertical asymptote.
	$y = \frac{x(x+1)+9}{x+1} = x + \frac{9}{x+1}$	**M1**	Finds oblique asymptote.
	$y = x$	**A1**	
		3	
7(b)	$\frac{dy}{dx} = 1 - 9(x+1)^{-2} = 0 \Rightarrow (x+1)^2 = 9$	**M1 A1**	Differentiates and sets derivative equal to 0.
	$(2, 5)$	**A1**	
	$(-4, -7)$	**A1**	
		4	

7(c)		**B1**	Axes labelled and correct asymptotes drawn.
		B1	Upper branch with $(0, 9)$ stated or shown on diagram.
		B1	Lower branch correct and good approach to asymptotes throughout, no extra branches.
		3	

7(d)		**B1 FT**	FT from sketch in **(c)** with asymptotes shown.
	$x^2+x+9=\dfrac{13}{2}(x+1)$ or $x^2+x+9=-\dfrac{13}{2}(x+1)$ $x^2-\frac{11}{2}x+\frac{5}{2}=0$ or $x^2+\frac{15}{2}x+\frac{31}{2}=0$	**M1 M1**	Finds critical points, award M1 for each case. May state that $x^2+x+9=-\frac{13}{2}(x+1)$ has no real solutions since $7>\frac{13}{2}$.
	$x=\frac{1}{2}, 5$	**A1**	
	$x<\frac{1}{2}$ and $x>5$.	**A1**	
		5	