Summation of series 1

Q1. Use the method of differences to find S_N, where

$$S_N = \sum_{n=1}^{N} \frac{1}{n(n+2)}.$$ [4]

Deduce the value of $\lim_{N\to\infty} S_N$. [1]

Q2. Express $\frac{1}{(2r+1)(2r+3)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^{n} \frac{1}{(2r+1)(2r+3)}.$$ [4]

Deduce the value of

$$\sum_{r=1}^{\infty} \frac{1}{(2r+1)(2r+3)}.$$ [1]

Q3. Given that $f(r) = \frac{1}{(r+1)(r+2)}$, show that

$$f(r-1) - f(r) = \frac{2}{r(r+1)(r+2)}.$$ [2]

Hence find $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$. [3]

Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}$. [1]

Q4. Use the method of differences to show that $\sum_{r=1}^{N} \frac{1}{(2r+1)(2r+3)} = \frac{1}{6} - \frac{1}{2(2N+3)}$. [5]

Deduce that $\sum_{r=N+1}^{2N} \frac{1}{(2r+1)(2r+3)} < \frac{1}{8N}$. [4]

Q5. Let $f(r) = r!(r-1)$. Simplify $f(r+1) - f(r)$ and hence find $\sum_{r=n+1}^{2n} r!(r^2+1)$. [5]

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Q6. Express $\dfrac{1}{r(r+1)(r-1)}$ in partial fractions. [1]

Find

$$\sum_{r=2}^{n} \frac{1}{r(r+1)(r-1)}.$$ [4]

State the value of

$$\sum_{r=2}^{\infty} \frac{1}{r(r+1)(r-1)}.$$ [1]

Q7. Expand and simplify $(r+1)^4 - r^4$. [1]

Use the method of differences together with the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2.$$ [4]

Q8. Use the List of Formulae (MF10) to show that $\sum_{r=1}^{13}(3r^2 - 5r + 1)$ and $\sum_{r=0}^{9}(r^3 - 1)$ have the same numerical value. [4]

Q9. Use the formula for $\tan(A - B)$ in the List of Formulae (MF10) to show that

$$\tan^{-1}(x+1) - \tan^{-1}(x-1) = \tan^{-1}\left(\frac{2}{x^2}\right).$$ [3]

Deduce the sum to n terms of the series

$$\tan^{-1}\left(\frac{2}{1^2}\right) + \tan^{-1}\left(\frac{2}{2^2}\right) + \tan^{-1}\left(\frac{2}{3^2}\right) + \dots .$$ [4]
