Summation of series 1 - MS

Q1. nth term is $\frac{1}{2}\left(\frac{1}{n}-\frac{1}{n+2}\right)$ M1A1

$$S_N = \frac{1}{2}\left[\begin{array}{l}\left(\frac{1}{N}-\frac{1}{N+2}\right)+\left(\frac{1}{N-1}-\frac{1}{N+1}\right)+\left(\frac{1}{N-2}-\frac{1}{N}\right)+\ldots \\ \left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{1}-\frac{1}{3}\right)\end{array}\right]$$ M1 sum of terms

$$= \frac{1}{2}\left[\frac{3}{2}-\frac{1}{N+2}-\frac{1}{N+1}\right]$$ A1 after cancellation [4]

Limit = ¾ B1√ [1]

Q2.

	Any method including cover-up rule.	$\frac{1}{(2r+1)(2r+3)}=\frac{1}{2}\left(\frac{1}{2r+1}-\frac{1}{2r+3}\right)$	B1		
	Expresses all terms as differences.	$S_n=\frac{1}{2}\left(\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)\ldots\left(\frac{1}{2n+1}-\frac{1}{2n+3}\right)\right)$	M1A1		
	Finds sum.	$=\frac{1}{6}-\frac{1}{2(2n+3)}$ (acf)	A1	4	
		$S_\infty=\frac{1}{6}$ (B0M1A1√ A0A1√ if signs reversed.)	A1	1	**[5]**

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Q3.

3	Proves initial result.	$f(r-1)-f(r)=\frac{1}{r(r+1)}-\frac{1}{(r+1)(r+2)}$	M1		
		$=\frac{r+2-r}{r(r+1)(r+2)}=\frac{2}{r(r+1)(r+2)}$ (AG)	A1	2	
	Sets up method of differences.	$\sum_1^n \frac{1}{r(r+1)(r+2)}=\frac{1}{2}\left\{\frac{1}{1\times 2}-\frac{1}{2\times 3}\right\}\ldots\ldots + \frac{1}{2}\left\{\frac{1}{n(n+1)}-\frac{1}{(n+1)(n+2)}\right\}$	M1A1		
	Shows cancellation to get result.	$=\frac{1}{4}-\frac{1}{2}\left\{\frac{1}{(n+1)(n+2)}\right\}$ (OE)	A1	3	
	States sum to infinity.	$\therefore \sum_1^\infty \frac{1}{r(r+1)(r+2)}=\frac{1}{4}$	A1√	1	[6]
	'Non hence' method for last two parts i.e. penalty of 1 mark.	$\frac{1}{r(r+1)(r-2)}=\frac{1}{2r}-\frac{1}{(r+1)}+\frac{1}{2(r+2)}$	(M1)		
		$\Rightarrow\cdots\Rightarrow$ $\frac{1}{2}-\frac{1}{2}+\frac{1}{4}\ldots+\frac{1}{2(n+1)}-\frac{1}{(n+1)}+\frac{1}{2(n+2)}$	(A1)		
		$=\frac{1}{4}-\frac{1}{2}\left\{\frac{1}{(n+1)(n+2)}\right\}$ (OE)	(A1)	(3)	
		$\therefore \sum_1^\infty \frac{1}{r(r+1)(r+2)}=\frac{1}{4}$	(A1√)	(1)	

Q4.

	Finds partial fractions.	$\frac{1}{(2r+1)(2r+3)}=\frac{1}{2}\left\{\frac{1}{2r+1}-\frac{1}{2r+3}\right\}$	M1A1		
		$\sum_{r=1}^{N}\frac{1}{(2r+1)(2r+3)}$			
	Expresses terms as differences.	$=\frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)+\ldots+\frac{1}{2}\left(\frac{1}{2N+1}-\frac{1}{2N+3}\right)$	M1A1		
	Shows cancellation.	$=\frac{1}{6}-\frac{1}{2(2N+3)}$ (AG)	A1	5	
	Uses $\sum_{N+1}^{2N}=\sum_{1}^{2N}-\sum_{1}^{N}$.	$\sum_{N+1}^{2N}=\left(\frac{1}{6}-\frac{1}{2(4N+3)}\right)-\left(\frac{1}{6}-\frac{1}{2(2N+3)}\right)$	M1		
	Applies result	$=\frac{1}{2}\left(\frac{1}{2N+3}-\frac{1}{4N+3}\right)$	A1		
	and simplifies.	$=\frac{N}{(2N+3)(4N+3)}$	M1		
	Deduces inequality.	$<\frac{N}{2N.4N}=\frac{1}{8N}$ (AG)	A1	4	[9]

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Q5.

	Simplifies.	$f(r+1) - f(r) = r(r+1)! - (r-1)r!$ $= r!(r^2 + r - r + 1) = r!(r^2+1)$	M1 A1		
	Uses difference method.	$\sum_{1}^{n} = f(2) - f(1) + f(3) - f(2) + \ldots f(n+1) - f(n)$ $= n(n+1)! - 0 = n(n+1)!$	M1 A1		
	Obtains result.	$\therefore \sum_{n+1}^{2n} = 2n(2n+1)! - n(n+1)!$ (Or directly using $\sum_{n+1}^{2n} = f(2n+1) - f(n+1)$ from the method of differences.)	A1	5	**[5]**

Q6.

	Finds partial fractions.	$\frac{1}{r(r-1)(r+1)} = \frac{1}{2(r-1)} - \frac{1}{r} + \frac{1}{2(r+1)}$	B1	**(1)**	
	Expresses each term in fractions	$\left(\frac{1}{2} - \frac{1}{2} + \frac{1}{6}\right) + \left(\frac{1}{4} - \frac{1}{3} + \frac{1}{8}\right) \ldots \left(\frac{1}{2(n-1)} - \frac{1}{n} + \frac{1}{2(n+1)}\right)$	M1A1		
	Cancels terms and sums	$= \frac{1}{4} - \frac{1}{2n} + \frac{1}{2(n+1)}$ (OE)	M1A1	**(4)**	
	Find sums to infinity	$S\infty = \frac{1}{4}$	B1	**(1)**	**[6]**

Q7.

	$(r+1)^4 - r^4 = 4r^3 + 6r^2 + 4r + 1$	B1 [1]
	$(n+1)^4 - 1^4 = 4\Sigma_{r=1}^{n} r^3 + 6\Sigma_{r=1}^{n} r^2 + 4\Sigma_{r=1}^{n} r + n$	M1
	$n^4 + 4n^3 + 6n^2 + 4n = 4\Sigma_{r=1}^{n} r^3 + n(2n^2 + 3n + 1) + 2n^2 + 2n + n$	A1A1
	$\Rightarrow \ldots \Rightarrow \Sigma_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$. (AG)	A1 [4]

Q8.

	$3 \times \frac{13 \times 14 \times 27}{6} - 5 \times \frac{13 \times 14}{2} + 13 = 2015$	M1A1
	$\left[\frac{9 \times 10}{2}\right]^2 - 10 = 2015$ (Award M1 for subtracting 9 or 10 here.)	M1A1 (4) **Total: 4**

Summation of series 1 - MS

Q9.		$A=\tan^{-1}(x+1)$; $B=\tan^{-1}(x-1) \Rightarrow \tan(A-B)=\dfrac{(x+1)-(x-1)}{1+(x+1)(x-1)}$ $\tan(A-B)=\dfrac{2}{x^2} \Rightarrow A-B=\tan^{-1}\left(\dfrac{2}{x^2}\right)$ (AG) LHS $=(\tan^{-1}2-\tan^{-1}0)+(\tan^{-1}3-\tan^{-1}1)+\ldots+(\tan^{-1}n-\tan^{-1}[n-2])$ $+(\tan^{-1}[n+1]-\tan^{-1}[n-1])$. $=\tan^{-1}[n+1]+\tan^{-1}n-\tan^{-1}1-\tan^{-1}0=\tan^{-1}[n+1]+\tan^{-1}n-\frac{1}{4}\pi$	M1A1 A1 (3) M1A1 A1A1 (4) **Total** **7**