Vectors 1

1. The lines l_1 and l_2 have vector equations

$$\mathbf{r} = 4\mathbf{i} - 2\mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 4\mathbf{i} - 5\mathbf{j} + 2\mathbf{k} + \mu(\mathbf{i} - \mathbf{j} - \mathbf{k})$$

respectively.

(i) Show that l_1 and l_2 intersect. [3]

(ii) Find the perpendicular distance from the point P whose position vector is $3\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}$ to the plane containing l_1 and l_2. [3]

(iii) Find the perpendicular distance from P to l_1. [4]

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2. The line l_1 passes through the point A whose position vector is $3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and is parallel to the vector $\mathbf{i} + \mathbf{j}$. The line l_2 passes through the point B whose position vector is $-\mathbf{i} - \mathbf{k}$ and is parallel to the vector $\mathbf{j} + 2\mathbf{k}$. The point P is on l_1 and the point Q is on l_2 and PQ is perpendicular to both l_1 and l_2.

(i) Find the length of PQ. [4]

(ii) Find the position vector of Q. [5]

(iii) Show that the perpendicular distance from Q to the plane containing AB and the line l_1 is $\sqrt{3}$. [4]

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3. The line l_1 passes through the point with position vector $8\mathbf{i} + 8\mathbf{j} - 7\mathbf{k}$ and is parallel to the vector $4\mathbf{i} + 3\mathbf{j}$. The line l_2 passes through the point with position vector $7\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and is parallel to the vector $4\mathbf{i} - \mathbf{k}$. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. In either order,

(i) show that $PQ = 13$,

(ii) find the position vectors of P and Q.

[9]

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4. The lines l_1 and l_2 have equations

$$l_1: \ \mathbf{r} = 6\mathbf{i} + 5\mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k}) \quad \text{and} \quad l_2: \ \mathbf{r} = 6\mathbf{i} + 5\mathbf{j} + 4\mathbf{k} + \mu(4\mathbf{i} + 6\mathbf{j} + \mathbf{k}).$$

Find a cartesian equation of the plane Π containing l_1 and l_2. [4]

Find the position vector of the foot of the perpendicular from the point with position vector $\mathbf{i} + 10\mathbf{j} + 3\mathbf{k}$ to Π. [4]

The line l_3 has equation $\mathbf{r} = \mathbf{i} + 10\mathbf{j} + 3\mathbf{k} + \nu(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$. Find the shortest distance between l_1 and l_3. [5]

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5. Find a cartesian equation of the plane Π containing the lines

$$\mathbf{r} = 3\mathbf{i} + \mathbf{k} + s(2\mathbf{i} + \mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 3\mathbf{i} - 7\mathbf{j} + 10\mathbf{k} + t(\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}).$$ [4]

The line l passes through the point P with position vector $6\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and is parallel to the vector $2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$. Find

(i) the position vector of the point where l meets Π, [3]

(ii) the perpendicular distance from P to Π, [3]

(iii) the acute angle between l and Π. [3]

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6. The position vectors of the points A, B, C, D are

$$2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}, \quad -2\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}, \quad \mathbf{i} + 4\mathbf{j} + \mathbf{k}, \quad \mathbf{i} + 5\mathbf{j} + m\mathbf{k},$$

respectively, where m is an integer. It is given that the shortest distance between the line through A and B and the line through C and D is 3. Show that the only possible value of m is 2. [7]

Find the shortest distance of D from the line through A and C. [3]

Show that the acute angle between the planes ACD and BCD is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$. [4]

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7. The plane Π has equation

$$\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) + \mu(3\mathbf{i} + \mathbf{j} - 2\mathbf{k}).$$

The line l, which does not lie in Π, has equation

$$\mathbf{r} = 3\mathbf{i} + 6\mathbf{j} + 12\mathbf{k} + t(8\mathbf{i} + 5\mathbf{j} - 8\mathbf{k}).$$

Show that l is parallel to Π. [4]

Find the position vector of the point at which the line with equation $\mathbf{r} = 5\mathbf{i} - 4\mathbf{j} + 7\mathbf{k} + s(2\mathbf{i} - \mathbf{j} + \mathbf{k})$ meets Π. [4]

Find the perpendicular distance from the point with position vector $9\mathbf{i} + 11\mathbf{j} + 2\mathbf{k}$ to l. [4]
