Vectors 2

1. The plane Π_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$. Find a cartesian equation of Π_1. [3]

The plane Π_2 has equation $2x - y + z = 10$. Find the acute angle between Π_1 and Π_2. [2]

Find an equation of the line of intersection of Π_1 and Π_2, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$. [5]

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2. The points A, B, C have position vectors

$$4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}, \qquad 5\mathbf{i} + 7\mathbf{j} + 8\mathbf{k}, \qquad 2\mathbf{i} + 6\mathbf{j} + 4\mathbf{k},$$

respectively, relative to the origin O. Find a cartesian equation of the plane ABC. [4]

The point D has position vector $6\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$. Find the coordinates of E, the point of intersection of the line OD with the plane ABC. [4]

Find the acute angle between the line ED and the plane ABC. [3]

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3. The line l_1 passes through the points $A\,(2, 3, -5)$ and $B\,(8, 7, -13)$. The line l_2 passes through the points $C\,(-2, 1, 8)$ and $D\,(3, -1, 4)$. Find the shortest distance between the lines l_1 and l_2. [5]

The plane Π_1 passes through the points A, B and D. The plane Π_2 passes though the points A, C and D. Find the acute angle between Π_1 and Π_2, giving your answer in degrees. [6]

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4. With respect to an origin O, the point A has position vector $4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and the plane Π_1 has equation

$$\mathbf{r} = (4 + \lambda + 3\mu)\mathbf{i} + (-2 + 7\lambda + \mu)\mathbf{j} + (2 + \lambda - \mu)\mathbf{k},$$

where λ and μ are real. The point L is such that $\overrightarrow{OL} = 3\overrightarrow{OA}$ and Π_2 is the plane through L which is parallel to Π_1. The point M is such that $\overrightarrow{AM} = 3\overrightarrow{ML}$.

(i) Show that A is in Π_1. [1]

(ii) Find a vector perpendicular to Π_2. [2]

(iii) Find the position vector of the point N in Π_2 such that ON is perpendicular to Π_2. [5]

(iv) Show that the position vector of M is $10\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}$ and find the perpendicular distance of M from the line through O and N, giving your answer correct to 3 significant figures. [6]

Vectors 2

5. The line l_1 is parallel to the vector $\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ and passes through the point A, whose position vector is $3\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$. The line l_2 is parallel to the vector $-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and passes through the point B, whose position vector is $-3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. Find

   (i) the length PQ, [5]

   (ii) the cartesian equation of the plane Π containing PQ and l_2, [4]

   (iii) the perpendicular distance of A from Π. [3]

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6. A line, passing through the point A (3, 0, 2), has vector equation $\mathbf{r} = 3\mathbf{i} + 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$. It meets the plane Π, which has equation $\mathbf{r}.(\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = 3$, at the point P. Find the coordinates of P. [3]

   Write down a vector $\mathbf{n}$ which is perpendicular to Π, and calculate the vector $\mathbf{w}$, where

   $$\mathbf{w} = \mathbf{n} \times (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}).$$ [3]

   The point Q lies in Π and is the foot of the perpendicular from A to Π. Use the vector $\mathbf{w}$ to determine an equation of the line PQ in the form $\mathbf{r} = \mathbf{u} + \mu\mathbf{v}$. [4]

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7. Find a cartesian equation of the plane Π_1 passing through the points with coordinates (2, −1, 3), (4, 2, −5) and (−1, 3, −2). [4]

   The plane Π_2 has cartesian equation $3x - y + 2z = 5$. Find the acute angle between Π_1 and Π_2. [3]

   Find a vector equation of the line of intersection of the planes Π_1 and Π_2. [4]

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8. The position vectors of the points A, B, C, D are

   $$\mathbf{a} = 2\mathbf{i} + \lambda\mathbf{j} - 3\mathbf{k}, \quad \mathbf{b} = 6\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}, \quad \mathbf{c} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}, \quad \mathbf{d} = \mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$$

   respectively. It is given that the shortest distance between the lines AB and CD is 3.

   (i) Show that $\lambda^2 + \lambda - 20 = 0$. [7]

   (ii) The planes p_1 and p_2 are the planes through A, B and D corresponding to the two values of λ satisfying the equation in part **(i)**. Find the acute angle between p_1 and p_2. [7]
