Equilibrium of a Rigid Body 2 MS

Q1.

	Volume	Centre of mass from AB
Small cone	$\frac{1}{3}\pi r^2.\frac{h}{2}$	$h+\frac{1}{4}.\frac{h}{2}\left(=\frac{9h}{8}\right)$
Large cone	$\frac{1}{3}\pi(3r)^2.\frac{3h}{2}$	$\frac{1}{4}.\frac{3h}{2}\left(=\frac{3h}{8}\right)$
Object	$\frac{26}{6}\pi(r)^2 h$	$\bar{x}$

4(a)		**B1**	For 9h/8 or 3h/8 (unsimplified)
	Take moments about AB $\frac{13}{3}\pi r^2 h.\bar{x}=\frac{27}{6}\pi r^2 h.\frac{3h}{8}-\frac{1}{6}\pi r^2 h.\frac{9h}{8}$	**M1 A1**	Moments equation: Allow use of relative masses 1, 26, 27
	$\bar{x}=\frac{9h}{26}$	**A1**	
		4	
4(b)	$\tan\theta=\frac{\bar{x}}{3r}$	**M1**	
	$\left(=\frac{3h}{26r}\right)$ Use $h=\frac{13}{4}r$ $\tan\theta=\frac{3}{8}$	**A1**	
		2	

Q2.

	Volume	Centre of mass from base
Cone	$\frac{1}{3}\pi(3r)^2.\ 4r$	$4r+r$
Cylinder	$\pi(3r)^2.\ 4r$	$2r$
Combined	$\frac{4}{3}\pi(3r)^2.\ 4r$	$\bar{x}$

3(a)		**B1**	Distances correct
	Taking moments about base of cylinder: $\bar{x}.\frac{4}{3}\pi(3r)^2.4r=\frac{1}{3}\pi(3r)^2.4r.5r+\pi(3r)^2.4r.2r$	**M1 A1**	Moments equation
	$\bar{x}=\frac{11}{4}r$	**A1**	
		4	
3(b)	Condition: $OG\cos\theta<OA$ (where O is vertex of cone and OA is slant height of cone)	**B1**	Correct condition for equilibrium
	$\left(4r+\frac{5r}{4}\right)\times\frac{4}{5}<5r$	**M1**	Expression in terms of r
	$21<25$ True	**A1**	Correct conclusion, with correct working
		3	

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Q3.

1	<table><tr><th></th><th>Area</th><th>Centre of mass from *DB*</th></tr><tr><td>*ABD*</td><td>$24a^2$</td><td>$-a$</td></tr><tr><td>*BCD*</td><td>$48a^2$</td><td>$2a$</td></tr><tr><td>Combined</td><td>$72a^2$</td><td>$\bar{x}$</td></tr></table>	**B1**	All distances correct. *ABCD* can be split in other ways, for example *ADC* and *ABC*.
	Taking moments about *DB*: $72a^2\bar{x} = 24a^2 \times -a + 48a^2 \times 2a$ *OR* Taking moments about *A*: $72a^2\bar{x} = 24a^2 \times 2a + 48a^2 \times 5a$ *OR* Taking moments about *G*: $24a^2(\bar{x}+a) = 48a^2 \times (2a-\bar{x})$	**M1**	Moments equation with masses in correct ratio.
	$\bar{x} = a$	**A1**	CWO
	Alternative method for question 1		
	ADC: distance of centre of mass from BD = $\dfrac{6a-3a}{3} = a$ *ABC*: distance of centre of mass from BD = $\dfrac{6a-3a}{3} = a$	**B1**	One calculation.
	Second calculation or statement about symmetry	**M1**	
	$\bar{x} = a$	**A1**	
		3	

Q4.

4(a)	<table><tr><th></th><th>Area</th><th>Centre of mass from *AD*</th></tr><tr><td>Square</td><td>$9a^2$</td><td>$\frac{3}{2}a$</td></tr><tr><td>*CDF*</td><td>$\frac{3}{2}ah$</td><td>a</td></tr><tr><td>*BEC*</td><td>$\frac{3}{2}ah$</td><td>$3a-\frac{1}{3}h$</td></tr><tr><td>Resulting *AEFC*</td><td>$9a^2-3ah$</td><td>$\bar{x}$</td></tr></table>	**M1**	Attempt at moments with three terms.
	Taking moments about *AD*: $(9a^2-3ah)\ \bar{x} = \left(9a^2 \times \frac{3}{2}a\right) - \left(\frac{3}{2}ah \times a\right) - \left(\frac{3}{2}ah \times \left(3a - \frac{1}{3}h\right)\right)$	**A1** **A1**	Two terms correct. All correct.
	$\bar{x} = \dfrac{27a^2 - 12ah + h^2}{6(3a-h)} \quad \left(= \dfrac{9a-h}{6}\right)$	**A1**	AEF
	$\bar{y} = \bar{x}$	**B1**	By symmetry or equal to their $\bar{x}$.
		5	
4(b)	For equilibrium, $\bar{x} \leqslant 3a - h$ $27a^2 - 12ah + h^2 \leqslant 6(3a-h)^2$	**B1**	Accept strict inequality.
	$27a^2 - 24ah + 5h^2 \geqslant 0$	**M1**	Homogeneous 3-term quadratic inequality.
	$h \leqslant \dfrac{9}{5}a$	**A1**	CAO.
		3	

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Q5.

4(a)	(table below)	M1	Attempt at moments, 3 terms.
	Taking moments about AB: $\left(\frac{2}{3}\pi a^3 - \pi ka\left(\frac{a}{2}\right)^2\right)\times\bar{x} = \left(\frac{2}{3}\pi a^3 \times \frac{3}{8}a\right) - \left(\pi ka\left(\frac{a}{2}\right)^2 \times \frac{ka}{2}\right)$	A1 A1	Any 2 terms correct. All correct.
	$\bar{x} = \frac{3a(2-k^2)}{2(8-3k)}$	A1	Shown convincingly, AG.
		4	
4(b)	$\tan\theta = \frac{\bar{x}}{a}$ $\frac{3(2-k^2)}{2(8-3k)} = \frac{7}{18}$	B1	
	$27k^2 - 21k + 2 = 0$	M1	Rearrange to form quadratic.
	$k = \frac{2}{3}$ and $k = \frac{1}{9}$	A1	Both answers correct.
		3	

Table in 4(a):

	Volume	Centre of mass from AB
Hemisphere	$\frac{2}{3}\pi a^3$	$\frac{3}{8}a$
Cylinder	$\pi ka(\frac{a}{2})^2$	$\frac{ka}{2}$
Remainder	$\frac{2}{3}\pi a^3 - \pi ka\left(\frac{a}{2}\right)^2$	$\bar{x}$

Q6.

7(a)	Frictional force = $\mu\times$ normal reaction at D and E	B1	$F_{AB} = \mu R,\ F_{BC} = \mu N$
	Moments about B, $Na - Ra = Wa(\sin\theta - \cos\theta)$ Moments about centre, $F_{AB}a + F_{BC}a = Wa(\cos\theta - \sin\theta)$ Moments about D, $F_{BC}a + Na = Wa(\cos\theta + \sin\theta)$ Moments about E, $Ra - F_{AB}a = Wa(\cos\theta + \sin\theta)$	B1	One moments equation about any point involving all relevant forces, resolved if necessary (AEF).
	Parallel to AB, $N - F_{AB} = W\sin\theta + W\sin\theta$ Perpendicular to AB, $F_{BC} + R = W\cos\theta + W\cos\theta$	B1	Two resolutions: all relevant terms, different frictional forces [Vertical: $R\cos\theta + F_{BC}\cos\theta + N\sin\theta = F_{AB}\sin\theta + W + W$ Horizontal: $F_{BC}\sin\theta + F_{AB}\cos\theta + R\sin\theta = N\cos\theta$] Alternative approach using two moments equations can earn the B1B1
	$N - R = \frac{1}{2}((1-\mu)N - (1+\mu)R)$	M1	Combine appropriate equations.
	$N\left(1-\frac{1}{2}(1-\mu)\right) = R\left(1-\frac{1}{2}(1+\mu)\right)$ $N\left(\frac{1}{2}+\frac{1}{2}\mu\right) = R\left(\frac{1}{2}-\frac{1}{2}\mu\right)$	M1	Collect terms to obtain ratio/fraction in terms of μ only (CWO), any equivalent simplified form.
	$R : N = 1+\mu : 1-\mu$	A1	
		6	

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Q7.

3(a)	Let F and R be friction and normal reaction at A Take moments about A, for rod $N\times 3a = W\times 2a\cos\theta + kW\times 4a\cos\theta$	**M1**	Correct terms, allow sign errors and cos/sin mix.
	$3N = (2+4k)W\times\frac{4}{5}$ $N = \frac{8}{15}W(1+2k)$	**A1**	At least one intermediate line of working. AG
		2	
3(b)	$\uparrow N\cos\theta + R = W + kW$	**B1**	Resolve (to include R) for rod.
	$\rightarrow F = N\sin\theta$ and $F = \frac{6}{7}R$	**B1**	Both.
	so $R = \frac{28}{75}W(1+2k)$ or $R = \frac{21}{45}W(1+k)$	**M1**	Find R or N.
	Eliminate to find k	**M1**	Complete method.
	$k = \frac{1}{3}$	**A1**	
		5	