Linear Motion Under a Variable Force 2 MS

Q1.

5	**(i)** $0.4v\,dv/dx = 0.4g\sin30 - 0.6x$	B1	Newton's Second Law, – sign essential
	$\int v\,dv = \int(5 - 1.5x)dx$	M1	Accept uncancelled integration
	$v^2/2 = 5x - 1.5x^2/2\ (+ c)$	A1	Accept omission of c
	$0.4g\sin30 - 0.6x = 0$	M1	Maximum speed when acceleration = 0
	$x = 3\frac{1}{3}$	A1	Accept 10/3
	$v^2/2 = 5 \times 10/3 - 1.5 \times (10/3)^2/2$	M1	
	$v = 4.08\ \text{ms}^{-1}$	A1 [7]	
	(ii) $0 = 5x - 1.5x^2/2$	M1	Uses $v = 0$ appropriately
	$x = 6\frac{2}{3} = 6.67$	A1 [2]	Not 20/3 **[9]**

Q2.

4	**(i)** $0.25dv/dt = -3t$	M1	Newton's Second Law, – sign essential
	$v = -12t^2/2\ (+ c)$	A1	Accept uncancelled form
	$0 = 12 \times 3^2/2 + c$	M1	Appropriate use of $v = 0$, $t = 3$
	Initial speed = $54\ \text{ms}^{-1}$	A1 [4]	Goes beyond $c = 54$
	(ii) $\int dx = \int(54 - 6t^2)dt$	M1	Separates variables, integrates v
	$x = [54t - 6t^3/3]_0^3$	A1√	√ candidates value [v in (i)]
	$x = 108\ \text{m}$	A1 [3]	**[7]**

Q3.

3	**(i)** $0.2v\,dv/dx = -0.4x$	M1	Newton's Second Law, – sign essential
	$v^2/2 = -2x^2/2\ (+ c)$	A1	Accept uncancelled form
	$0 = -2 \times 2.5^2/2 + c \rightarrow c = 6.25$	M1	
	KE = $0.2 \times 6.25 = 1.25\ \text{J}$	A1 [4]	$v = 3.54\ \text{ms}^{-1}$
	(ii) $2^2/2 = -2x^2/2 + 6.25$	M1	$v = 2$ in accurate integral attempt at limits or finding arbitrary constant e.g. in (i)
	$x = 2.06$	A1 [2]	**[6]**

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Q4.

3	(i)	$0.2\,\mathrm{d}v/\mathrm{d}t = 0.2g - 0.8v$	M1		Use Newton's Second Law, – sign essential
		$a = (\mathrm{d}v/\mathrm{d}t =)10 - 4v$ AG	A1	[2]	
	(ii)	$\int 1/(10-4v)\,\mathrm{d}v = \int \mathrm{d}t$	M1		Separates variables and attempts to integrate
		$\frac{-1}{4}\ln(10-4v) = t\,(+c)$	A1		
		$[c = \frac{-1}{4}\ln 10]$	M1		Attempts to find the constant or uses the correct limits
		$\frac{-1}{4}\ln(10-4v) = 0.6 - \frac{1}{4}\ln 4$	A1		
		$v = 2.27$	A1	[5]	

Q5.

6 (i)	$0.4\mathrm{d}v/\mathrm{d}t = \mathrm{T}–0.4g \times 0.5–0.9v$	B1		Not awarded for N2L round corner
	$0.2\mathrm{d}v/\mathrm{d}t= 0.2g–\mathrm{T}–0.9v$	B1		Not awarded for N2L round corner
	$0.6\mathrm{d}v/\mathrm{d}t=0.2g–0.4g \times 0.5–1.8v$	M1		Awarded for N2L round corner
	$\mathrm{d}v/\mathrm{d}t = -3v$ AG	A1	[4]	

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(ii)	$\int dv/v = \int -3dt$	M1		Separates variables, integrates
	$\ln v = -3t\ (+\ c)$	A1		Accurare integrals
	$c = \ln 5$	B1		Or $[\ln v]_5^{2.5} = [-3t]_0^t$ implied
	t = 0.231	A1		(ln2)/3
	$\int dx = \int e^{c-3t}\,dt$	M1		Attempts integration of v(t)
	$x = -[e^{c-3t}]_0^{0.231}/3$	A1		Correct integral and limits
	$x = 0.833$ m	A1	[7]	5/6 m
	OR			
	$v\,dv/dx = -3v$, $dv/dx = -3$ $\int dv = \int -3dx$	M1		Attempts integration
	$[v]_5^{2.5} = [-3x]_0^x$	A1		Correct integral and limits
	$x = 0.833$m	A1		Accept 5/6m

Q6.

4	**(i)**	a = 10 – 0.45v AG	B1	[1]	0.2a = 0.2g – 0.09v or similar should be seen	
	(ii)	$\int 1/(10-0.45v)dv = \int dt$	M1		An attempt at integration needed	
		–ln(10 – 0.45v)/0.45 = t (+c)	A1			
		t = 0, v = 4, c = –4.67(58..)	DM1		Attempts to find c or uses correct limits	
		–ln(10 – 0.45v)/0.45 = 1.5 – 4.676	M1		Uses t = 1.5 and evaluated c	
		v = 12.9 or 13.0	A1	[5]		**[6]**

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Q7.

7 **(i)**	$0.5a = 0.16e^{x}$	M1		N2L, single force	
	$a = 0.32e^{x}$	A1			
	$\int v\,dv = \int 0.32e^{x}\,dx$	M1		Forms integral from $v\,dv/dx = a$	
	$v^2/2 = 0.32e^{x}$ (+c)	A1		Award if c omitted	
	$x = 0$, $v = 0.8$ hence c = 0, so $v^2 = 0.64e^{x}$	M1		Trying to find the value of c	
	$v = 0.8e^{x/2}$ AG	A1	[6]		
OR	$dv/dt = 0.8e^{x/2}\,x\,dx/dt$	M1		Uses chain rule on given answer	
	$dv/dt = 0.4e^{x/2}.v$	A1		Maybe implied by later work	
	$x = 0$, $v = 0.8e^{0}$	M1		Finding speed where $x = 0$	
	$x = 0$, $v = 0.8$	A1			
	$0.5dv/dt = (0.2e^{x/2})(0.8e^{x/2})$	M1		Expresses "ma" in terms of x	
	$0.5acc^{n} = 0.16e^{x}$	A1			
(ii)	$\int e^{-x/2}\,dx = \int 0.8\,dt$	M1		Forms integral from $dx/dt = 0.8e^{x/2}$	
	$e^{-x/2}/(-1/2) = 0.8t$ (+c)	A1		Award if c omitted	
	$x = 0$, $t = 0$, hence c = –2 and $-2e^{-1.4/2} = 0.8t - 2$	M1		Finding c and using $x = 1.4$ or $[e^{-x/2}/(-1/2)]_0^{1.4} = 0.8t$	
	$t = 1.26$ s	A1	[4]	1.2585..	**[10]**

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Q8.

3 **(i)**	$0.8v\mathrm{d}v/\mathrm{d}x = 4\mathrm{e}^{-x} - 2.4x^2$	M1	N2L, terms different signs	
	$v\mathrm{d}v/\mathrm{d}x = 5\mathrm{e}^{-x} - 3x^2$ AG	A1 [2]		
(ii)	$\int v\mathrm{d}v = \int (5\mathrm{e}^{-x} - 3x^2)\mathrm{d}x$	M1	Attempts integration	
	$v^2/2 = -5\mathrm{e}^{-x} - 3x^3/3\ (+\mathrm{c})$	A1	Accept c omitted	
	$x = 0,\ v = 6$, hence c = 23	B1	Or uses limits 0 and 2	
	$v^2/2 = -5\mathrm{e}^{-2} - 3x2^3/3 + 23$	M1	Puts $x = 2$ in $v(x)$ expression	
	$v = 5.35\ \mathrm{ms}^{-1}$	A1 [5]	$v = 5.352..$	**[7]**

Q9.

3 **(i)**	$0.2a = 0.42$ $v = (2.1 \times 1) = 2.1$	M1 A1	[2]	Newton's Second Law and $v = u + at$
(ii)	$0.2\mathrm{d}v/\mathrm{d}t = 0.42 - 0.32t$ $[v]_{2.1}^{v} = [2.1\mathrm{t} - 0.8\mathrm{t}^2]_1^2$ $v = 1.8$	M1 M1 A1	[3]	Newton's Second Law with $\mathrm{a} = \mathrm{d}v/\mathrm{d}t$
(iii)	$v = \int (0.42 - 0.32t + 0.06\mathrm{t}^2)\mathrm{d}t / 0.2$ $v = [0.42t - 0.16t^2 + 0.02t^3]_0^3 / 0.2$ or $[v]_{1.8}^{v} = [0.42t - 1.16t^2 + 0.02t^3]_2^3 / 0.2$ $v = 1.8$, so no change	M1 M1 A1	[3]	For attempt to integrate and correct limits seen