Linear Motion Under a Variable Force 2

Q1.

A particle P of mass 0.4 kg is released from rest at the top of a smooth plane inclined at $30°$ to the horizontal. The motion of P down the slope is opposed by a force of magnitude $0.6x$ N, where x m is the distance P has travelled down the slope. P comes to rest before reaching the foot of the slope. Calculate

(i) the greatest speed of P during its motion, [7]

(ii) the distance travelled by P during its motion. [2]

Q2.

A particle P of mass 0.25 kg moves in a straight line on a smooth horizontal surface. At time t s the velocity of P is v m s^{-1}. A variable force of magnitude $3t$ N opposes the motion of P.

(i) Given that P comes to rest when $t = 3$, find v when $t = 0$. [4]

(ii) Calculate the distance travelled by P in the interval $0 \leqslant t \leqslant 3$. [3]

Q3.

A particle P of mass 0.2 kg is projected horizontally from a fixed point O, and moves in a straight line on a smooth horizontal surface. A force of magnitude $0.4x$ N acts on P in the direction PO, where x m is the displacement of P from O.

(i) Given that P comes to instantaneous rest when $x = 2.5$, find the initial kinetic energy of P. [4]

(ii) Find the value of x on the first occasion when the speed of P is 2 m s^{-1}. [2]

Q4.

A particle P of mass 0.2 kg is released from rest and falls vertically. At time t s after release P has speed v m s^{-1}. A resisting force of magnitude $0.8v$ N acts on P.

(i) Show that the acceleration of P is $(10 - 4v)$ m s^{-2}. [2]

(ii) Find the value of v when $t = 0.6$. [5]

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Q5.

Two particles P and Q, of masses 0.4 kg and 0.2 kg respectively, are attached to opposite ends of a light inextensible string. P is placed on a horizontal table and the string passes over a small smooth pulley at the edge of the table. The string is taut and the part of the string attached to Q is vertical (see diagram). The coefficient of friction between P and the table is 0.5. Q is projected vertically downwards with speed 5 m s^{-1}, and at time t s after the instant of projection the speed of the particles is $v\text{ m s}^{-1}$. The motion of each particle is opposed by a resisting force of magnitude $0.9v$ N. The particle P does not reach the pulley.

(i) Show that $\frac{dv}{dt} = -3v$. [4]

(ii) Find the value of t when the particles have speed 2.5 m s^{-1} and the distance that each particle has travelled in this time. [7]

Q6.

A particle of mass 0.2 kg is projected vertically downwards with initial speed 4 m s^{-1}. A resisting force of magnitude $0.09v$ N acts vertically upwards on the particle during its descent, where $v\text{ m s}^{-1}$ is the downwards velocity of the particle at time t s after being set in motion.

(i) Show that the acceleration of the particle is $(10 - 0.45v)\text{ m s}^{-2}$. [1]

(ii) Find v when $t = 1.5$. [5]

Q7.

A particle P of mass 0.5 kg moves in a straight line on a smooth horizontal surface. The velocity of P is $v\text{ m s}^{-1}$ when the displacement of P from O is x m. A single horizontal force of magnitude $0.16e^x$ N acts on P in the direction OP. The velocity of P when it is at O is 0.8 m s^{-1}.

(i) Show that $v = 0.8e^{\frac{1}{2}x}$. [6]

(ii) Find the time taken by P to travel 1.4 m from O. [4]

Linear Motion Under a Variable Force 2

Q8.

A particle P of mass 0.8 kg moves along the x-axis on a horizontal surface. When the displacement of P from the origin O is x m the velocity of P is v m s^{-1} in the positive x-direction. Two horizontal forces act on P. One force has magnitude $4e^{-x}$ N and acts in the positive x-direction. The other force has magnitude $2.4x^2$ N and acts in the negative x-direction.

(i) Show that $v\frac{dv}{dx} = 5e^{-x} - 3x^2$. [2]

(ii) The velocity of P as it passes through O is 6 m s^{-1}. Find the velocity of P when $x = 2$. [5]

Q9.

A small block B of mass 0.2 kg is placed at a fixed point O on a smooth horizontal surface. A horizontal force of magnitude 0.42 N is applied to B. At time t s after the force is first applied, the velocity of B away from O is v m s^{-1}.

(i) Find the value of v when $t = 1$. [2]

For $t > 1$ an additional force, of magnitude $0.32t$ N and directed towards O, is applied to B. The force of magnitude 0.42 N continues to act as before.

(ii) Find the value of v when $t = 2$. [3]

For $t > 2$ a third force, of magnitude $0.06t^2$ N and directed away from O, is applied to B. The other two forces continue to act as before.

(iii) Show that the velocity of B is the same when $t = 2$ and when $t = 3$. [3]
