Linear Motion Under a Variable Force 1 MS

Q1.

6(i)	$R = 0.2g + 0.4t\sin\theta$ $(= 2 + 0.24t)$ $F = 0.5(2 + 0.24t) = 1 + 0.12t$	M1	Note $\sin\theta = 0.6$ and $\cos\theta = 0.8$ $(\theta = 36.87°)$ Resolve vertically and use $F = \mu R$
	$0.4t\cos\theta = 1 + 0.12t$	M1	Resolve horizontally
	$t = 5$	A1	
		3	
6(ii)	$0.2\mathrm{d}v/\mathrm{d}t = 0.4t \times 0.8 - (1 + 0.12t)$	M1	Use Newton's Second Law horizontally
	$\mathrm{d}v / \mathrm{d}t = t - 5$ **AG**	A1	
		2	

6(iii)	$\int \mathrm{d}v = \int (t-5)\mathrm{d}t$ $v = t^2/2 - 5t + c$	M1	Attempt to integrate the equation from part **(ii)**
	$v = 0$ when $t = 5$ hence $c = 12.5$	A1	Finds the constant of integration, c
	$v = 8^2/2 - 5 \times 8 + 12.5 = 4.5$	A1	Find v when $t = 8$
	$a = -0.5 \times 0.2g / 0.2 = -5\ \mathrm{m\,s^{-1}}$ and $s = 4.5^2/(2 \times 5)$	M1	Finds a and uses $v^2 = u^2 + 2as$
	$s = 2.025$ m	A1	
		5	

Q2.

3(i)	$0.4\dfrac{\mathrm{d}v}{\mathrm{d}t} = 0.8t - 2e^{-t}$	M1	Use Newton's Second Law horizontally
	$\dfrac{\mathrm{d}v}{\mathrm{d}t} = 2t - 5e^{-t}$	A1	**AG**
		2	
3(ii)	$\int dv = \int (2t - 5e^{-t})\mathrm{d}t$ $v = t^2 + 5e^{-t} (+\ \mathrm{c})$	M1	Attempt to integrate the equation from part **(i)**
	$t = 1$ and $v = 8$ so c = 5.16	M1	Attempt to find the constant of integration, c
	$v = t^2 + 5e^{-t} + 5.16$ or $v = t^2 + 5e^{-t} + 7 - 5e^{-1}$	A1	
		3	
3(iii)	Evaluates v for $t = 0$	M1	
	$V = 10.2\ \mathrm{ms^{-1}}$	A1	
		2	

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Q3.

7(i)	$0.2\text{d}v/\text{d}t = 0.2\,g + 0.6t - k\,e^{-t}$	M1	Use Newton's Second Law downwards
	$\text{d}v/\text{d}t = 10 + 3t - 5\,ke^{-t}$ **AG**	A1	
	Total:	2	
7(ii)	$\text{d}v/\text{d}t = 10 - 5k\,e^{0} = 0$	M1	Recognise that dv/dt = 0 when t = 0
		M1	Attempts to solve the equation
7(ii)	k = 2	A1	
	Total:	3	

7(iii)	$\int dv = \int(10 + 3t - 5k\,e^{-t})\text{d}t$	M1	Attempts to integrate the equation from part i with k not replaced
	$[v = 10t + 3t^2/2 + 5e^{-t} + c,\ v = 0,\ t = 0 \text{ so } c = -5]$ $v = 10t + 3t^2/2 + 5e^{-t} - 5$	A1	
	$\int dx = \int(10t + 3t^2/2 + 5e^{-t} - 5)\text{d}t$ $x = 5t^2 + t^3/2 - 5e^{-t} - 5t + c$	M1	Attempts to integrate again. Allow their k or just k not replaced
	$x = 0,\ t = 0$, so $c = 5$ and substitutes $t = 2$ $x = 5 \times 2^2 + 2^3/2 - 5e^{-2} - 5 \times 2 + 5$	M1	
	Height = 18.3 m	A1	
	Total:	5	

Q4.

4(i)	$T = 16(1.6 - 0.8 - x)/0.8\ (= 16 - 20x)$	B1	Use $T = \lambda x/L$
	$0.5v\text{d}v/\text{d}x = 16(1.6 - 0.8 - x)/0.8 - 48x^2$	M1	Use Newton's Second Law horizontally
	$v\text{d}v/\text{d}x = 32 - 40x - 48x^2$ **AG**	A1	
		3	

4(ii)	$48x^2 + 40x - 32 = 0$	M1	Put acceleration = 0 for maximum velocity
	$x = 0.5$	A1	
	$\int v dv = \int (32 - 40x - 48x^2)\text{d}x$ $(v^2/2 = 32x - 40x^2/2 - 48x^3/3 + c)$	M1	Attempt to integrate the equation from part **(i)**
	$4.5^2/2 = 32\times0.5 - 20 \times0.5^2 - 16\times0.5^3 + c,\ c = 1.125$	M1	Substitute $x = 0.5$, $v = 4.5$ to find c
	$v = 1.5$	A1	Use $x = 0$
		5	

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Q5.

7(i)	$0.2mg = 0.06 \times 8$	M1	Resolve along the plane
	$m = 0.24$ kg **AG**	A1	
		2	
7(ii)	$m\frac{dv}{dt} = 0.06t - 0.2mg$ or $0.24\frac{dv}{dt} = 0.06t - 0.2 \times 0.24g$	M1	Use N2L along the plane
	$\frac{dv}{dt} = 0.25t - 2$ **AG**	A1	
	$\int dv = \int(0.25t - 2)dt$	M1	Attempt to integrate
	$v = 0.25t^2/2 - 2t + c$, Put $v = 0$ and $t = 4$ (leads to $c = 6$)	M1	Attempt to find c
	Initial velocity $= 6 \text{ m s}^{-1}$	A1	
		5	

Q6.

3(i)	$0.25v\frac{dv}{dx} = -kv^2x^{-2} \to v\frac{dv}{dx} = -4kv^2x^{-2}$	B1	AG
		1	
3(ii)	$\int\frac{dv}{v} = -4k\int x^{-2}dx$	M1	Attempt to integrate
	$\ln v = \frac{4k}{x}(+c)$	A1	
	$x = 0.8, v = 3$ hence $c = \ln 3 - 5k$	A1	Finds c
	$\ln v = \frac{4k}{x} + \ln 3 - 5k$	M1	
	$v = 3e^{\left(\frac{4k}{x} - 5k\right)}$	A1	
		5	

Q7.

6(i)	$0.2v\frac{dv}{dx} = 0.09\sqrt{x} - 0.3$	M1	Use Newton's Second Law horizontally
	$v\frac{dv}{dx} = 0.45\sqrt{x} - 1.5$	A1	AG
		2	
6(ii)	$0 = 0.45x^{\frac{1}{2}} - 1.5$	M1	Equate acceleration to zero
	$x = \frac{100}{9}$	A1	
		2	

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6(iii)	$\int v\,dv = \int(0.45x^{\frac{1}{2}} - 1.5)dx$	M1	Attempt to integrate
	$\frac{v^2}{2} = \frac{0.45^{\frac{3}{2}}}{\frac{3}{2}} - 1.5x(+c) = 0.3x^{\frac{3}{2}} - 1.5x(+c)$	A1	
	$0.3\left(\frac{100}{9}\right)^{\frac{3}{2}} - 1.5\left(\frac{100}{9}\right) + c = 0$	M1	
	$c = \frac{50}{9}$	A1	
	$x = 0,\ \frac{v^2}{2} > \frac{50}{9}$ so $v > \frac{10}{3}$	A1	
		5	

Q8.

5(a)	$\frac{dv}{3u - v} = k\,dt$	M1
	$-\ln(3u - v) = kt + d$ $t = 0, v = u: \quad d = -\ln 2u$	M1
	$v = 2u: \quad t = \frac{1}{k}\ln 2$	A1
		3
5(b)	$v\frac{dv}{dx} = 3ku - kv \ \left[\Rightarrow \frac{v\,dv}{3u - v} = k\,dx\right]$	B1
	$\frac{(-(3u - v) + 3u)dv}{3u - v} = k\,dx$ so $-v - 3u\ln(3u - v) = kx + c$	M1A1
	$x = 0, v = u: \ c = -u - 3u\ln 2u$	M1
	$v = 2u: \ x = \frac{u}{k}(3\ln 2 - 1)$	A1
		5

Q9.

7(a)	$v\frac{dv}{dx} = -\frac{100}{x^3} + \frac{200}{x^2}$ $\frac{v^2}{2} = \frac{50}{x^2} - \frac{200}{x} + A$	M1 A1	Correct equation and attempt to integrate Correct
	$x = 1, v = -10: \quad A = 200$	M1	Use initial condition
	$v^2 = \frac{100(2x - 1)^2}{x^2}$	M1	Rearrange to find v^2
	$v = \pm\frac{10(2x - 1)}{x}$ and take negative sign to meet initial condition, so $v = \frac{10(1 - 2x)}{x}$	A1	Convincingly shown (no mention of $\pm$ scores A0) AG
		5	

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7(b)	$\frac{x\,\mathrm{d}x}{1-2x}=10\mathrm{d}t$ $\frac{1}{2}\left(\frac{1}{1-2x}-1\right)\mathrm{d}x=10\mathrm{d}t$ $-\frac{1}{4}\ln\lvert 1-2x\rvert-\frac{x}{2}=10t+B$	**M1 A1**	Rearrange and attempt to integrate
	$t=0, x=1:\quad B=-\frac{1}{2}$	**M1**	Use initial condition
	$2x-2=-40t-\ln(\lvert 1-2x\rvert)$ so $e^{-40t}=(2x-1)e^{2x-2}$	**A1**	Convincingly shown, working required AG
	For large values of t, $x\to\frac{1}{2}$	**B1**	CAO
		5	