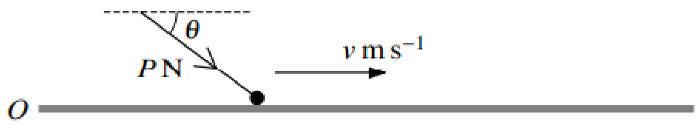
Linear Motion Under a Variable Force 1

Q1.



A small object of mass 0.2 kg rests at a point O on a rough horizontal surface. The coefficient of friction between the object and the surface is 0.5. A force of magnitude P N acting at an angle θ below the horizontal is applied to the object. The velocity of the object is $v\ \text{m s}^{-1}$ away from O at time t s after the force begins to act (see diagram). It is given that $\tan\theta = \frac{3}{4}$ and that $P = 0.4t$ for $0 \leqslant t \leqslant 8$.

(i) Find the value of t when the object starts to move. [3]

(ii) Show that, when the force is acting and the object is in motion, $\frac{\mathrm{d}v}{\mathrm{d}t} = t - 5$. [2]

When $t = 8$ the force of magnitude P N ceases to act.

(iii) Find the distance travelled by the object after $t = 8$ before it comes to rest. [5]

Q2.

A particle P of mass 0.4 kg is projected horizontally along a smooth horizontal plane from a point O. At time t s after projection the velocity of P is $v\ \text{m s}^{-1}$. A force of magnitude $0.8t$ N directed away from O acts on P and a force of magnitude $2\mathrm{e}^{-t}$ N opposes the motion of P.

(i) Show that $\frac{\mathrm{d}v}{\mathrm{d}t} = 2t - 5\mathrm{e}^{-t}$. [2]

(ii) Given that $v = 8$ when $t = 1$, express v in terms of t. [3]

(iii) Find the speed of projection of P. [2]

Q3.

A particle P of mass 0.2 kg is released from rest at a point O above horizontal ground. At time t s after its release the velocity of P is $v\ \text{m s}^{-1}$ downwards. A vertically downwards force of magnitude $0.6t$ N acts on P. A vertically upwards force of magnitude $k\mathrm{e}^{-t}$ N, where k is a constant, also acts on P.

(i) Show that $\frac{\mathrm{d}v}{\mathrm{d}t} = 10 - 5k\mathrm{e}^{-t} + 3t$. [2]

(ii) Find the greatest value of k for which P does not initially move upwards. [3]

(iii) Given that $k = 1$, and that P strikes the ground when $t = 2$, find the height of O above the ground. [5]

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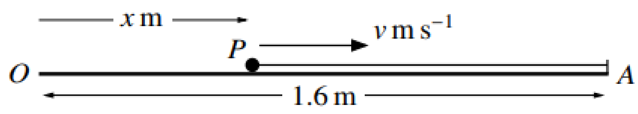
Q4.



A particle P of mass 0.5 kg is projected along a smooth horizontal surface towards a fixed point A. Initially P is at a point O on the surface, and after projection, P has a displacement from O of x m and velocity $v\,\text{m s}^{-1}$. The particle P is connected to A by a light elastic string of natural length 0.8 m and modulus of elasticity 16 N. The distance OA is 1.6 m (see diagram). The motion of P is resisted by a force of magnitude $24x^2$ N.

(i) Show that $v\dfrac{dv}{dx} = 32 - 40x - 48x^2$ while P is in motion and the string is stretched. [3]

The maximum value of v is 4.5.

(ii) Find the initial value of v. [5]

Q5.

A particle P is projected horizontally from a point O on a rough horizontal surface. The coefficient of friction between the particle and the surface is 0.2. A horizontal force of magnitude $0.06t$ N directed away from O acts on P, where t s is the time after projection. P comes to rest when $t = 4$.

(i) The particle begins to move again when $t = 8$. Show that the mass of P is 0.24 kg. [2]

(ii) Show that, for $0 \leqslant t \leqslant 4$, $\dfrac{dv}{dt} = 0.25t - 2$, and find the speed of projection of P. [5]

(iii) Find the distance from O at which P comes to rest. [4]

Q6.

A smooth horizontal surface has two fixed points O and A which are 0.8 m apart. A particle P of mass 0.25 kg is projected with velocity $3\,\text{m s}^{-1}$ horizontally from A in the direction away from O. The velocity of P is $v\,\text{m s}^{-1}$ when the displacement of P from O is x m. A force of magnitude kv^2x^{-2} N opposes the motion of P.

(i) Show that $v\dfrac{dv}{dx} = -4kv^2x^{-2}$. [1]

(ii) Express v in terms of k and x. [5]

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Q7.

A particle P of mass 0.2 kg is projected horizontally from a fixed point O on a smooth horizontal surface. When the displacement of P from O is x m the velocity of P is $v\,\text{m}\,\text{s}^{-1}$. A horizontal force of variable magnitude $0.09\sqrt{x}$ N directed away from O acts on P. An additional force of constant magnitude 0.3 N directed towards O acts on P.

(i) Show that $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 0.45\sqrt{x} - 1.5$. [2]

(ii) Find the value of x for which the acceleration of P is zero. [2]

(iii) Given that the minimum value of v is positive, find the set of possible values for the speed of projection. [5]

Q8.

A particle P is moving along a straight line with acceleration $3ku - kv$ where v is its velocity at time t, u is its initial velocity and k is a constant. The velocity and acceleration of P are both in the direction of increasing displacement from the initial position.

(a) Find the time taken for P to achieve a velocity of $2u$. [3]

(b) Find an expression for the displacement of P from its initial position when its velocity is $2u$. [5]

Q9.

A particle P moving in a straight line has displacement x m from a fixed point O on the line at time t s. The acceleration of P, in $\text{m}\,\text{s}^{-2}$, is given by $\dfrac{200}{x^2} - \dfrac{100}{x^3}$ for $x > 0$. When $t = 0$, $x = 1$ and P has velocity $10\,\text{m}\,\text{s}^{-1}$ directed towards O.

(a) Show that the velocity $v\,\text{m}\,\text{s}^{-1}$ of P is given by $v = \dfrac{10(1-2x)}{x}$. [5]

(b) Show that x and t are related by the equation $\mathrm{e}^{-40t} = (2x-1)\mathrm{e}^{2x-2}$ and deduce what happens to x as t becomes large. [5]
