Motion of a Projectile 2 MS

Q1.

5(a)	Quote trajectory equation from MF19 and use $\cos\theta = 1/\sec\theta$ $y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$	B1	Must include step with $\sec^2\theta$ Allow derived from first principles AG
		1	

5(b)	$16 = 20 - \frac{10\times100}{2u^2}(1+4)$	M1	Substitute into result **(a)**
	$u^2 = 625, \quad (u = 25)$	A1	
	Use equation again: $30 = 18\tan\theta - \frac{10\times324}{2\times625}\left(1+(\tan\theta)^2\right)$	M1	
	$2.592(\tan\theta)^2) - 18\tan\theta + 32.592 = 0$	A1	3 term quadratic. Alternatives include: $54t^2 - 375t + 679 = 0$, $324t^2 - 2250t + 4074 = 0$
	Discriminant = $324 - 4\times2.592\times32.592 = -13.91$	M1	Discriminant for alternatives: -6039 and -217404
	As this is less than 0, no real solutions for θ	A1	CWO
		6	

Q2.

7(a)	At greatest height $0 = 100\sin\theta - gt$	M1	
	$t = 8$	A1	
	Therefore times at height H are $t = 3$ (and $t = 13$)	B1	
	Substitute into $H = 100\sin\theta t - \frac{1}{2}gt^2$	M1	
	$H = 195$	A1	
	Alternative method to question **7(a)**		
	$\uparrow H = 100\sin\theta t - \frac{1}{2}gt^2$	M1	
	And $H = 100\sin\theta(t+10) - \frac{1}{2}g(t+10)^2$	A1	
	Subtract: $1000\sin\theta = \frac{1}{2}g(20t+100)$	M1	
	$t = 3$	B1	
	$H = 195$	A1	

7(a)	Alternative method to question **7(a)**		
	$\uparrow H = 100\sin\theta t - \frac{1}{2}gt^2$	B1	
	Difference between roots = $\frac{\sqrt{(100\sin\theta)^2 - 2gH}}{\frac{1}{2}g}$	M1 A1	
	Equate to 10 and rearrange to find H	M1	
	$H = 195$	A1	
		5	

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7(b)	Time to required point = 15 s	B1	
	$\uparrow v = 100\sin\theta - 10\times15 (= -70)$ $\rightarrow v = 100\cos\theta = 60$	B1	Both components.
	Magnitude = 92.2	B1	
	Angle below horizontal = $\tan^{-1}(70/60) = 49.4°$	B1	
		4	

Q3.

7(a)	$y = 0$ in trajectory equation: $R\tan\theta - g\dfrac{R^2}{2u^2(\cos\theta)^2} = 0$	M1	
	$(R =)\dfrac{2u^2\sin\theta\cos\theta}{g}$ only	A1	Any equivalent single term expression, for example: $\dfrac{u^2\sin 2\theta}{g}$, $\dfrac{2u^2\tan\theta}{g\sec^2\theta}$, at least one intermediate line of working, not just quoting a result. SC B1 using SUVAT.
		2	
7(b)	$x = their\dfrac{u^2\sin\theta\cos\theta}{g}$ and substitute in trajectory equation.	M1	Or use SUVAT.
	$H = \dfrac{u^2(\sin\theta)^2}{2g}$	A1	Single term.
		2	
7(c)	Use $R = \dfrac{4H}{\sqrt{3}}$ and simplify: $\tan\theta = \sqrt{3}$, $\theta = 60°$	B1	AG
		1	

Q4.

5	At A: $\uparrow u\sin\theta - 8g$ $\quad\rightarrow u\cos\theta$	M1	Both.
	$\tan\alpha = \dfrac{u\sin\theta - 8g}{u\cos\theta}$	A1	
	At B: $\uparrow u\sin\theta - 32g$ $\quad\rightarrow u\cos\theta$	M1	Both.
	$\tan\beta = \dfrac{u\sin\theta - 32g}{u\cos\theta}$	A1	
	$\dfrac{u\sin\theta - 8g}{u\cos\theta}\times\dfrac{u\sin\theta - 32g}{u\cos\theta} = -1$	B1	Perpendicular directions, so $\tan\alpha\times\tan\beta = -1$.
	$u^2 - 320u + 25600 = 0$	M1	Simplify to a quadratic in u.
	$u = 160$	A1	
		7	

Q5.

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1(a)	Velocity: $\rightarrow u\cos\alpha$	**B1**	
	$\uparrow u\sin\alpha - gT$	**B1**	Allow 10 for g. Must be T.
		2	
1(b)	$\frac{u\cos\alpha}{u\sin\alpha - gT} = -\frac{\sin\alpha}{\cos\alpha}$ oe	**M1 FT**	Allow missing minus sign on RHS for M1. FT from **(a)**.
	$T = \frac{u}{g\sin\alpha}$	**A1**	
		2	
1(c)	$\sin\alpha < 1$ giving $T > \frac{u}{g}$	**B1**	AG
		1	

Q6.

7(a)	Coordinates of A: $x = a\sin 60, \quad y = a - a\cos 60$	**B1**	
	$\frac{a}{2} = \frac{a\sqrt{3}}{2}\sqrt{3} - \frac{g\frac{(a\sqrt{3})^2}{2^2}}{2V^2 . \frac{1}{4}}$	**M1**	Substitute *their* (x, y) into correct trajectory equation.
	Rearrange to find V^2.	**M1**	
	$V^2 = \frac{3}{2}ag, \quad V = \sqrt{\frac{3}{2}ag}$	**A1**	
		4	
7(b)	$\frac{1}{2}mu^2 - \frac{1}{2}mV^2 = mga(1 + \cos 60)$	**M1**	Energy equation.
	$u^2 = \frac{9}{2}ag$	**A1**	u is the speed at P.
	$T - mg = \frac{m}{a}u^2$	**M1**	N2L
	$T = \frac{11}{2}mg$	**A1**	
		4	

Q7.

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7(a)	For Q: $x = u\cos\beta T$	B1	
	For P: $x = \frac{35}{2}\cos\alpha(T+1)$	B1	
	Collision, so $\frac{35}{2}\cos\alpha(T+1) = u\cos\beta T$	M1	Equate and attempt to rearrange.
	$\frac{35}{2}\times\frac{3}{5}(T+1) = u\times\frac{2}{\sqrt{5}}T$ $4uT = 21\sqrt{5}(T+1)$	A1	AG Shown convincingly.
		4	
7(b)	Vertical motion to collision: For Q: $y = u\sin\beta T - \frac{1}{2}gT^2$ For P: $y = \frac{35}{2}\sin\alpha(T+1) - \frac{1}{2}g(T+1)^2$	M1 A1	M1 for both expressions, one correct.
	Equate: $u\times\frac{1}{\sqrt{5}}T - \frac{1}{2}gT^2 = \frac{35}{2}\times\frac{4}{5}(T+1) - \frac{1}{2}g(T+1)^2$ $14(T+1) - \frac{1}{2}g(T^2+2T+1-T^2) = \frac{21}{4}(T+1)$	M1	Equate and attempt to solve
	$16T + 36 = 21T + 21$, $15 = 5T$ $T = 3$	A1	
		4	

7(c)	$x = 42$	B1	
	$\|y\| = 24$	M1	
	$y = -24$ (or 24 m below O)	A1	Correct sign or in words.
		3	

Q8.

3(a)	Components of velocity : $\rightarrow 25\cos\theta$ $\uparrow 25\sin\theta - 2g$	B1	
	Speed = $\sqrt{(25\cos\theta)^2 + (25\sin\theta - 2g)^2}$	M1 A1	Expression for speed or square of speed.
	$(25\cos\theta)^2 + (25\sin\theta - 2g)^2 = 15^2$ $625 - 100g\sin\theta + 4g^2 = 225$	M1	Attempt to solve and find value for $\sin\theta$
	$\sin\theta = \frac{800}{1000} = \frac{4}{5}$	A1	
		5	

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3(b)	Time of flight $= \left(\dfrac{2\times 25\sin\theta}{g}\right) = 4$ (s)	**B1**	
	Range $= \dfrac{2\times 25\sin\theta}{g}\times 25\cos\theta$	**M1**	Any equivalent method.
	Range = 60 (m)	**A1**	CWO
	Alternative method for question 3(b)		
	$y = \dfrac{4}{3}x - \dfrac{1}{45}x^2$	**B1**	Equation of trajectory..
	Substitute $y = 0$ and solve	**M1**	
	60 (m)	**A1**	
		3	