Motion of a Projectile 1 M

Q1.

6(i)	**Alternative Method**		
	$h = (15\sin\theta)\times 4 - \frac{g(4)^2}{2}$	**B1**	
	$\frac{m(15)^2}{2} = \frac{m(30)^2}{2} + mgh$	**M1**	Allow h not replaced
		M1	Attempt to eliminate h and attempt to solve for θ
	$\theta = 50.4°$	**A1**	
		4	

6(ii)	$s = 15\sin 50.4\times 4 - \frac{1}{2}\times g\times 4^2$	**M1**	Use vertical motion. Allow *their* θ for first M1
	$s = 33.75$ m **AG**	**A1**	
	$\cos\alpha = \frac{15\cos 50.4}{30}$	**M1**	Use trigonometry of a right angled triangle
	α = 71.4° below the horizontal	**A1**	
		4	If g = 9.8 or 9.81 used then M1A0M1A0

Q2.

2(i)	$-15\sin\theta = 15\sin\theta - 2g$	**M1**	Use $v = u + at$ vertically
	$(\theta =)\ 41.8$	**A1**	
		2	
2(ii)	Vertically: $\frac{v}{15\cos\theta} = \pm\tan 20$	**M1**	v = vertical velocity
	$v = (\pm)4.07$	**A1**	
	$-4.07 = 15\sin 41.8 - gt$	**M1**	Use $v = u + at$ vertically
	$(t =)$ 1.41 s	**A1**	
		4	

Motion of a Projectile 1 M

Q3.

4(i)	$x = 30\cos 60t$	**B1**	Use horizontal motion
	$y = 30\sin 60t - \frac{gt^2}{2}$	**B1**	Use s = ut + $\frac{gt^2}{2}$ vertically
	$y = \frac{30\sin 60x}{30\cos 60} - \frac{5x^2}{(30\cos 60)^2}$	**M1**	Attempt to eliminate t
	$y = 1.73x - 0.0222x^2$ or $y = \sqrt{3}x - \frac{x^2}{45}$	**A1**	
		4	
4(ii)	$x = y$ or $\tan 45 = \frac{y}{x}$	**M1**	
	$1 = 1.73 - 0.0222x$ or $1 = \sqrt{3} - \frac{x}{45}$	**M1**	x common to all three terms
	$x = 32.9$	**A1**	
		3	

Q4.

2(i)	$V\cos 30 = 40$	**M1**	Note V is the velocity of projection
	$V = 46.2\text{ m }s^{-1}$	**A1**	Allow $\frac{80}{\sqrt{3}}$ or $\frac{80\sqrt{3}}{3}$
	$y = 23.1t - 5t^2$	**B1FT**	Use $s = ut + \frac{at^2}{2}$ vertically. FT candidates half V but not $V = 40$ used
		3	
2(ii)	$y = \frac{23.1x}{40} - \frac{5x^2}{1600}$	**M1**	Attempt to eliminate t by substituting $t = \frac{x}{40}$ into answer to **part (i)**
	$y = 0.577x - \frac{x^2}{320}$ or $y = 0.577x - 0.003125x^2$	**A1**	
		2	

Q5.

4(i)	Velocity component vertically = $\pm(V\sin 60 - 3g)$	**B1**	Use $v = u + at$
	$\tan 30 = \frac{30 - V\sin 60}{V\cos 60}$	**M1**	Use trigonometry of a right angled triangle
	$V = 15\sqrt{3} = 26(.0)\text{ m s}^{-1}$	**A1**	
		3	

4(ii)	$y = 26\sin 60 \times 3 - \frac{g \times 3^2}{2}$	**B1FT**	Use $s = ut + \frac{at^2}{2}$ vertically. Their V from **part (i)**
	$D^2 = (26\sin 60 \times 3 - g \times 3^2)^2 + (26\cos 60 \times 3)^2$	**M1**	Use Pythagoras's Theorem
	$D = 45(.0)\text{ m}$	**A1**	
		3	

Motion of a Projectile 1 M

Q6.

1	For greatest height, $T = \frac{u}{2g}$	B1
	At $t = \frac{2T}{3}, \uparrow v_v = \frac{u}{2} - \frac{2Tg}{3} = \frac{u}{6}$	M1
	$\rightarrow v_h = \frac{u\sqrt{3}}{2}$	A1
	Speed = $\sqrt{v_v^2 + v_h^2}$ = $\sqrt{\frac{u^2}{36} + \frac{3u^2}{4}}$	M1
	$= \frac{\sqrt{7}}{3}u$	A1
		5

Q7.

6(a)	Greatest height = $\frac{(u\sin\theta)^2}{2g}$	M1A1
	At ¾ greatest height, $\rightarrow v\cos\alpha = u\cos\theta$	M1
	At ¾ greatest height, $\uparrow (v\sin\alpha)^2 = (u\sin\theta)^2 - 2g.\frac{3}{4}\frac{(u\sin\theta)^2}{2g}$	M1
	$v\sin\alpha = \frac{1}{2}u\sin\theta$	A1
	So $\tan\alpha = \frac{1}{2}\tan\theta$ **AG**	A1
		6

Q8.

5(a)	$\rightarrow x = u\cos\alpha t$ $\uparrow y = u\sin\alpha t - \frac{1}{2}gt^2$	B1	Both
	Eliminate t: $y = u\sin\alpha.\frac{x}{u\cos\alpha} - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$	M1	Eliminate
	$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha$	A1	AG
		3	
5(b)	Greatest height = $\frac{(u\sin\alpha)^2}{2g} = \frac{u^2}{4g}$	M1 A1	Accept alternative methods, for example differentiate expression in **(a)** and equate to 0.
	$t = u\sin 45/g$ so $d = u\cos 45.u\sin 45/g = \frac{u^2}{2g}$	A1	AG
		3	

Motion of a Projectile 1 M

5(c)	Use greatest height displacements in trajectory equation $\frac{u^2}{4g}=\frac{u^2}{2g}\tan\alpha-\frac{gu^4}{2u^2 4g^2}\sec^2\alpha$	**M1**	Use equation of trajectory (substitute coordinates of Q
	$u^2=2u^2\tan\alpha-\frac{u^2}{2}(1+\tan^2\alpha)$	**M1**	Use of $\sec^2\alpha=\left(1+\tan^2\alpha\right)$
	$\tan^2\alpha-4\tan\alpha+3=0$	**M1**	Obtain a three-term quadratic in $\tan\alpha$
	$\tan\alpha=1,\ 3$ so $\alpha=71.6°$	**A1**	Both solutions needed
		4	