

Discrete Random Variables 1

Q1.

The probability distribution of the discrete random variable X is shown in the table below.

x	-3	-1	0	4
$P(X = x)$	a	b	0.15	0.4

Given that $E(X) = 0.75$, find the values of a and b . [4]

Q2.

A small farm has 5 ducks and 2 geese. Four of these birds are to be chosen at random. The random variable X represents the number of geese chosen.

- (i) Draw up the probability distribution of X . [3]
- (ii) Show that $E(X) = \frac{8}{7}$ and calculate $\text{Var}(X)$. [3]
- (iii) When the farmer's dog is let loose, it chases either the ducks with probability $\frac{3}{5}$ or the geese with probability $\frac{2}{5}$. If the dog chases the ducks there is a probability of $\frac{1}{10}$ that they will attack the dog. If the dog chases the geese there is a probability of $\frac{3}{4}$ that they will attack the dog. Given that the dog is not attacked, find the probability that it was chasing the geese. [4]
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Q3.

Set A consists of the ten digits 0, 0, 0, 0, 0, 0, 2, 2, 2, 4.

Set B consists of the seven digits 0, 0, 0, 0, 2, 2, 2.

One digit is chosen at random from each set. The random variable X is defined as the sum of these two digits.

- (i) Show that $P(X = 2) = \frac{3}{7}$. [2]
- (ii) Tabulate the probability distribution of X . [2]
- (iii) Find $E(X)$ and $\text{Var}(X)$. [3]
- (iv) Given that $X = 2$, find the probability that the digit chosen from set A was 2. [2]
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Discrete Random Variables 1

Q4.

Sanket plays a game using a biased die which is twice as likely to land on an even number as on an odd number. The probabilities for the three even numbers are all equal and the probabilities for the three odd numbers are all equal.

(i) Find the probability of throwing an odd number with this die. [2]

Sanket throws the die once and calculates his score by the following method.

- If the number thrown is 3 or less he multiplies the number thrown by 3 and adds 1.
- If the number thrown is more than 3 he multiplies the number thrown by 2 and subtracts 4.

The random variable X is Sanket's score.

(ii) Show that $P(X = 8) = \frac{2}{9}$. [2]

The table shows the probability distribution of X .

x	4	6	7	8	10
$P(X = x)$	$\frac{3}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

(iii) Given that $E(X) = \frac{58}{9}$, find $\text{Var}(X)$. [2]

Sanket throws the die twice.

(iv) Find the probability that the total of the scores on the two throws is 16. [2]

(v) Given that the total of the scores on the two throws is 16, find the probability that the score on the first throw was 6. [3]

Q5.

The discrete random variable X takes the values 1, 4, 5, 7 and 9 only. The probability distribution of X is shown in the table.

x	1	4	5	7	9
$P(X = x)$	$4p$	$5p^2$	$1.5p$	$2.5p$	$1.5p$

Find p . [3]

Discrete Random Variables 1

Q6.

- (i) State three conditions that must be satisfied for a situation to be modelled by a binomial distribution. [2]

On any day, there is a probability of 0.3 that Julie's train is late.

- (ii) Nine days are chosen at random. Find the probability that Julie's train is late on more than 7 days or fewer than 2 days. [3]
- (iii) 90 days are chosen at random. Find the probability that Julie's train is late on more than 35 days or fewer than 27 days. [5]
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Q7.

In a probability distribution the random variable X takes the value x with probability kx , where x takes values 1, 2, 3, 4, 5 only.

- (i) Draw up a probability distribution table for X , in terms of k , and find the value of k . [3]
- (ii) Find $E(X)$. [2]
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Q8.

The possible values of the random variable X are the 8 integers in the set $\{-2, -1, 0, 1, 2, 3, 4, 5\}$. The probability of X being 0 is $\frac{1}{10}$. The probabilities for all the other values of X are equal. Calculate

- (i) $P(X < 2)$, [2]
- (ii) the variance of X , [3]
- (iii) the value of a for which $P(-a \leq X \leq 2a) = \frac{17}{35}$. [1]
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Q9.

A biased die was thrown 20 times and the number of 5s was noted. This experiment was repeated many times and the average number of 5s was found to be 4.8. Find the probability that in the next 20 throws the number of 5s will be less than three. [4]

Discrete Random Variables 1

Q10.

Judy and Steve play a game using five cards numbered 3, 4, 5, 8, 9. Judy chooses a card at random, looks at the number on it and replaces the card. Then Steve chooses a card at random, looks at the number on it and replaces the card. If their two numbers are equal the score is 0. Otherwise, the smaller number is subtracted from the larger number to give the score.

(i) Show that the probability that the score is 6 is 0.08. [1]

(ii) Draw up a probability distribution table for the score. [2]

(iii) Calculate the mean score. [1]

If the score is 0 they play again. If the score is 4 or more Judy wins. Otherwise Steve wins. They continue playing until one of the players wins.

(iv) Find the probability that Judy wins with the second choice of cards. [3]

(v) Find an expression for the probability that Judy wins with the n th choice of cards. [2]
