Coordinate Geometry 2 MS

Q1.

3 $mx+14=\frac{12}{x}+2 \rightarrow mx^2+12x-12=0$ Uses $b^2=4ac \rightarrow m=-3$ $-3x^2+12x-12=0 \rightarrow P(2, 8)$ [Or $m=-12x^{-2}$ M1 Sub M1 $x=2$ A1] [$\rightarrow m=-3$ and $y=8$ M1 A1]	M1 M1 A1 DM1 A1 [5]	Eliminates x (or y) Any use of discriminant Any valid method.

Q2.

7 $3y+2x=33.$ Gradient of line = −⅔ Gradient of perpendicular = 3/2 Eqn of perp $y-3=\frac{3}{2}(x+1)$ Sim Eqns $\rightarrow$ (3, 9) (−1, 3) → (3, 9) → (7, 15)	B1 M1 M1 M1 A1 M1 A1 [7]	 Use of $m_1m_2=-1$ with gradient of line Correct form of perpendicular eqn. Sim eqns. Vectors or other method.

Q3.

7	A (2, 14), B (14, 6) and C (7, 2).			
(i)	m of $AB=-⅔$	B1		
	m of perpendicular $=\frac{3}{2}$	M1		For use of $m_1m_2=-1$
	eqn of AB $y-14=-\frac{2}{3}(x-2)$	M1		Allow M1 for unsimplified eqn
	eqn of CX $y-2=\frac{3}{2}(x-7)$	M1		Allow M1 for unsimplified eqn
	Sim Eqns $\rightarrow$ X (11, 8)	M1 A1	[6]	For solution of sim eqns.
(ii)	$AX : XB = 14-8 : 8-6 = 3 : 1$ Or $\sqrt{(9^2+6^2)} : \sqrt{(3^2+2^2)} = 3 : 1$	M1 A1	[2]	Vector steps or Pythagoras.

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Q4.

7	(i) mid-point = (3, 4) Grad. $AB = -\frac{1}{2} \rightarrow$ grad. of perp., = 2 $y - 4 = 2(x - 3)$ $y = 2x - 2$	**B1** **M1** **M1** **A1** **[4]**	soi For use of $-1/m$ soi ft on *their* (3, 4) and 2
	(ii) $q = 2p - 2$ √ $\quad p^2 + q^2 = 4$ oe $p^2 + (2p-2)^2 = 4 \rightarrow 5p^2 - 8p = 0$ $\{\text{OR } \frac{1}{4}(q+2)^2 + q^2 = 4 \rightarrow 5q^2 + 4q - 12 = 0\}$ $(0, -2)$ and $\left(\frac{8}{5}, \frac{6}{5}\right)$	**B1**√ **B1** **M1** **A1A1** **[5]**	ft for 1st eqn. Attempt substn (linear into quadratic) & simplify

Q5.

5(i)	Eqn of AC $y = -\frac{1}{2}x + 4$ (gradient must be $\Delta y / \Delta x$)	**M1A1**	Uses gradient and a given point for equa. CAO
	Gradient of $OB = 2 \rightarrow y = 2x$ (If y missing only penalise once)	**M1 A1**	Use of $m_1m_2 = -1$, answers only ok.
		4	

5(ii)	Simultaneous equations $\rightarrow$ ((1.6, 3.2))	**M1**	Equate and solve for M1 and reach $\geqslant 1$ solution
	This is mid-point of OB. $\rightarrow B$ (3.2, 6.4)	**M1 A1**	Uses mid-point. CAO
	or		
	Let coordinates of B (h, k) $OA = AB \rightarrow h^2 = 8k - k^2$ $OC = BC \rightarrow k^2 = 16h - h^2 \rightarrow (3.2, 6.4)$		M1 for both equations, M1 for solving with $y = 2x$
	or		
	gradients $\left(\frac{k-4}{h} \times \frac{k}{h-8} = -1\right)$		M1 for gradient product as -1, M1 solving with $y = 2x$
	or		
	Pythagoras: $h^2 + (k-4)^2 + (h-8)^2 + k^2 = 4^2 + 8^2$		M1 for complete equation, M1 solving with $y = 2x$
		3	

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Q6.

8	**EITHER**		
	Gradient of bisector $= -\frac{3}{2}$	**B1**	
	gradient $AB = \frac{5h-h}{4h+6-h}$	***M1**	Attempt at $\frac{y-step}{x-step}$
	Either $\frac{5h-h}{4h+6-h} = \frac{2}{3}$ or $-\frac{4h+6-h}{5h-h} = -\frac{3}{2}$	***M1**	Using $m_1m_2 = -1$ appropriately to form an equation.
	OR		
	Gradient of bisector $= -\frac{3}{2}$	**B1**	
	Using gradient of AB and A, B or midpoint $\rightarrow \frac{2}{3}x + \frac{h}{3} = y$ oe	***M1**	Obtain equation of AB using gradient from $m_1m_2 = -1$ and a point.
	Substitute co-ordinates of one of the other points	***M1**	Arrive at an equation in h.
	h = 2	**A1**	
	Midpoint is $\left(\frac{5h+6}{2}, 3h\right)$ or (8, 6)	**B1FT**	Algebraic expression or FT for numerical answer from '*their h*'
	Uses midpoint and '*their h*' with $3x + 2y = k$	**DM1**	Substitutes '*their midpoint*' into $3x + 2y = k$. If $y = -\frac{3}{2}x + c$ is used (expect $c = 18$) the method mark should be withheld until they ×2.
	$\rightarrow k = 36$ soi	**A1**	
		7	

Q7.

7(a)	$r^2\left[=(5-2)^2+(7-5)^2\right]=13$	**B1**	$r^2 = 13$ or $r = \sqrt{13}$
	Equation of circle is $(x-5)^2+(y-2)^2=13$	**B1 FT**	OE. FT on *their* 13 but LHS must be correct.
		2	
7(b)	$(x-5)^2 + (5x-10-2)^2 = 13$	**M1**	Substitute $y = 5x-10$ into *their* equation.
	$26x^2-130x+156$ $[=0]$	**A1 FT**	OE 3-term quadratic with all terms on one side. FT on *their* circle equation.
	$[26](x-2)(x-3)$ $[=0]$	**M1**	Solve 3-term quadratic in x by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of x^2.
	(2, 0), (3, 5)	**A1 A1**	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two x or y values only. If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two x or y values only.
	$(AB)^2 = (3-2)^2+(5-0)^2$	**M1**	SOI. Using *their* points to find length of AB.
	$AB = \sqrt{26}$	**A1**	ISW. Dependent on final M1 only.

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7(b)	**Alternative method for question 7(b)**		
	$\left(\frac{y+10}{5}-5\right)^2+(y-2)^2=13$	**M1**	Substitute $x=\frac{y+10}{5}$ into *their* equation.
	$\frac{26y^2}{25}-\frac{26y}{5}\ [=0]$	**A1 FT**	OE 2-term quadratic with all terms on one side. FT on *their* circle equation.
	$[26]y(y-5)\ [=0]$	**M1**	Solve 2-term quadratic in y by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of y^2.
	(2, 0), (3, 5)	**A1 A1**	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two x or y values only. If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two x or y values only.
	$(AB)^2=(3-2)^2+(5-0)^2$	**M1**	SOI. Using *their* points to find length of AB.
	$AB=\sqrt{26}$	**A1**	ISW. Dependent on final M1 only.
		7	

Q8.

9(a)	$x^2+(2x+5)^2=20$ leading to $x^2+4x^2+20x+25=20$	**M1**	Substitute $y=2x+5$ and expand bracket.
	$(5)(x^2+4x+1)[=0]$	**A1**	3-term quadratic.
	$x=\frac{-4\pm\sqrt{16-4}}{2}$	**M1**	OE. Apply formula or complete the square.
	$A=(-2+\sqrt{3},1+2\sqrt{3})$	**A1**	Or 2 correct x values.
	$B=(-2-\sqrt{3},1-2\sqrt{3})$	**A1**	Or all values correct. **SC B1** all 4 values correct in surd form without working. **SC B1** all 4 values correct in decimal form from correct formula or completion of the square
	$AB^2=their(x_2-x_1)^2+their(y_2-y_1)^2$	**M1**	Using *their* coordinates in a correct distance formula. Condone one sign error in x_2-x_1 or y_2-y_1
	$\left[AB^2=48+12 \text{ leading to}\right] AB=\sqrt{60}$	**A1**	OE. CAO. Do not accept decimal answer. Answer must come from use of surd form in distance formula.
		7	

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9(b)	$x^2+m^2(x-10)^2=20$	*M1	Finding equation of tangent and substituting into circle equation.
	$x^2(m^2+1)-20m^2x+20(5m^2-1)$ $[=0]$	DM1	OE. Brackets expanded and all terms collected on one side of the equation.
	$[b^2-4ac=]400m^4-80(m^2+1)(5m^2-1)$	M1	Using correct coefficients from *their* quadratic equation.
	$400m^4-80(5m^4+4m^2-1)=0 \rightarrow (-80)(4m^2-1)=0$	A1	OE. Must have '=0' for A1.
	$m=\pm\frac{1}{2}$	A1	
	Alternative method for question 9(b)		
	Length, l of tangent, is given by $l^2=10^2-20$	M1	
	$l=\sqrt{80}$	A1	
	$\tan\alpha=\frac{\sqrt{20}}{\sqrt{80}}=\frac{1}{2}$	M1 A1	Where α is the angle between the tangent and the x-axis.
	$m=\pm\frac{1}{2}$	A1	
		5	

Q9.

6(a)	$(x+1)^2+(3x-22)^2=85$	M1	OE. Substitute equation of line into equation of circle.
	$10x^2-130x+400$ $[=0]$	A1	Correct 3-term quadratic
	$[10](x-8)(x-5)$ leading to $x=8$ or 5	A1	Dependent on factors or formula or completing of square seen.
	(8, 4), (5, –5)	A1	If M1A1A0A0 scored, then **SC B1** for correct final answer only.
		4	
6(b)	Mid-point of $AB=\left(6\frac{1}{2},-\frac{1}{2}\right)$	M1	Any valid method
	Use of $C=(-1, 2)$	B1	SOI
	$r^2=\left(-1-6\frac{1}{2}\right)^2+\left(2+\frac{1}{2}\right)^2$	M1	Attempt to find r^2. Expect $r^2=62\frac{1}{2}$.
	Equation of circle is $(x+1)^2+(y-2)^2=62\frac{1}{2}$	A1	OE.
		4	

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Q10.

9(a)	Express as $(x+3)^2+(y-1)^2=26+9+1[=36]$	**M1**	Completing the square on x and y or using the form $x^2+y^2+2gx+2fy+c=0$, centre $(-g,-f)$ and radius $\sqrt{g^2+f^2-c}$. SOI by correct answer.
	Centre (−3, 1)	**B1**	
	Radius 6	**B1**	
	So lowest point is (−3, −5)	**A1 FT**	FT on *their* centre and *their* radius.
		4	
9(b)	Intersects when $x^2+(kx-5)^2+6x-2(kx-5)-26=0$ **or** $(x+3)^2+(kx-5-1)^2=36$	***M1**	Substituting $y=kx-5$ into *their* circle equation or rearranging and equating y.
	$x^2+k^2x^2-10kx+25+6x-2kx+10-26=0$ **or** $x^2+6x+9+k^2x^2-12kx+36=36$ leading to $k^2x^2+x^2+6x-12kx+9[=0]$ **or** $(k^2+1)x^2+(6-12k)x+9[=0]$	**DM1** **A1**	Rearranging to 3-term quadratic (terms grouped, all on one side). Allow 1 error. Correct quadratic (need to see 9 as constant term).
	$(6-12k)^2-4(k^2+1)\times 9\ [>0]$ [leading to $144k^2-144k+36-36k^2-36>0$]	**DM1**	Using discriminant $b^2-4ac[>0]$ with *their* values. Allow if in square root.
	[$108k^2-144k=0$ leading to] $k=0$ or $k=\frac{4}{3}$	**A1**	Need not see method for solving.
	$k<0, k>\frac{4}{3}$	**A1**	Do not accept $\frac{4}{3}<k<0$.
		6	