Differential Equations 1 MS

Q1.

8	Forms and solves AQE.	$m^2+2m+5=0 \Rightarrow m=-1\pm 2\mathrm{i}$	M1		
	States CF	CF $\mathrm{e}^{-t}(A\cos 2t+B\sin 2t)$ (OE)	A1		
	States form for PI.	PI $x=p\cos t+q\sin t \Rightarrow \dot{x}=-p\sin t+q\cos t$ $\Rightarrow \ddot{x}=-p\cos t-q\sin t$	M1		
	Substitutes in equation.	$-p\cos t-q\sin t-2p\sin t+2q\cos t$ $+5p\cos t+5q\sin t=10\sin t$			
	Obtains values for p and q by comparing coefficients.	$4p+2q=0$ and $-2p+4q=10$	M1		
		$\Rightarrow p=-1,\ q=2$	A1		
	States GS.	GS $x=\mathrm{e}^{-t}(A\cos 2t+B\sin 2t)+2\sin t-\cos t$ (OE)	A1	6	
	Uses initial conditions to evaluate constants.	$t=0\quad x=5 \Rightarrow A=6$	B1		
		$\dot{x}=-\mathrm{e}^{-t}(A\cos 2t+B\sin 2t)$ $+\mathrm{e}^{-t}(-2A\sin 2t+2B\cos 2t)+2\cos t+\sin t$	M1		
		$2=-6+2B+2 \Rightarrow B=3$	A1		
	States particular solution.	$x=\mathrm{e}^{-t}(6\cos 2t+3\sin 2t)+2\sin t-\cos t$ (OE)	A1	4	
	Gives req. approximate solution.	As $t\to\infty\quad x\approx 2\sin t-\cos t$ (The final mark is independent of A and B).	B1	1	**[11]**

Q2.

8	Forms AQE and solves.	$m^2+2m+5=0 \Rightarrow m=-1\pm 2\mathrm{i}$	M1A1		
	Writes CF.	CF: $y=\mathrm{e}^{-x}(A\cos 2x+B\sin 2x)$	A1		
	Correct form for PI and differentiates twice.	$y=k\mathrm{e}^{-2x} \Rightarrow y'=-2k\mathrm{e}^{-2x} \Rightarrow y''=4\mathrm{e}^{-2x}$	M1		
	Substitutes.	$\Rightarrow 4k-4k+5k=10 \Rightarrow k=2$	M1		
	Writes PI.	PI: $y=2\mathrm{e}^{-2x}$	A1		
	Writes GS.	GS: $y=\mathrm{e}^{-x}(A\cos 2x+B\sin 2x)+2\mathrm{e}^{-2x}$	A1		
	Uses $y(0)=5$ to find A.	$y=5, x=0 \Rightarrow 5=A+2 \Rightarrow A=3$	B1		
	Uses $y'(0)=1$ to find B.	$y'=-\mathrm{e}^{-x}(A\cos 2x+B\sin 2x)$ $+\mathrm{e}^{-x}(-2A\sin 2x+2B\cos 2x)-4\mathrm{e}^{-2x}$	M1		
		$y'=1, x=0 \Rightarrow 1=-3+2B-4 \Rightarrow B=4$	A1		
	Writes particular solution.	$\therefore\ y=\mathrm{e}^{-x}(3\cos 2x+4\sin 2x)+2\mathrm{e}^{-2x}$	A1 CAO	11	**[11]**

Q3.

10(i)	$y'=x+x't$	**B1**	
	$y''=x'+x'+tx''$	**B1**	
	Substitute correctly	**B1**	AG
10(ii)	Auxiliary equation: $m^2+9=0 \Rightarrow m=\pm 3i$.	**M1**	Correct auxiliary
	CF $=A\cos 3t+B\sin 3t$.	**A1**	

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	PI $y = at^2+bt+c$ so $y' = 2at + b$ and $y'' = 2a$	M1	Differentiate twice and substitute
	$a = \frac{1}{3}, b = 0, c = \frac{1}{27}$.	A1	
	$y = A\cos 3t + B\sin 3t + \frac{1}{3}t^2 + \frac{1}{27}$	A1FT	Their CF + their PI both in correct form
	$x = \frac{\pi}{9}$ when $t = \frac{\pi}{3}$ gives $A = \frac{1}{27}$.	B1	
	$y' = -3A\sin 3t + 3B\cos 3t + \frac{2}{3}t$	M1	Differentiating their y of equivalent difficulty
	$x' = \frac{2}{3}$ when $t = \frac{\pi}{3}$ gives $B = -\frac{\pi}{27}$.	A1	
	$x = \frac{\cos 3t - \pi\sin 3t + 9t^2 + 1}{27t}$	A1	AEF
		12	

Q4.

1	$e^{\int 5\mathrm{d}x} = e^{5x}$	M1 A1
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(ye^{5x}\right) = e^{-2x}$	M1
	$ye^{5x} = -\frac{1}{2}e^{-2x} + C$	A1
	$0 = -\frac{1}{2} + C$	M1
	$y = \frac{1}{2}e^{-5x} - \frac{1}{2}e^{-7x}$	A1
		6

Q5.

7(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = t^3\frac{\mathrm{d}y}{\mathrm{d}t} + 3t^2y$	B1
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = t^3\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 6t^2\frac{\mathrm{d}y}{\mathrm{d}t} + 6ty$	B1
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 13x = t^3\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 6t^2\frac{\mathrm{d}y}{\mathrm{d}t} + 6ty + 4t^3\frac{\mathrm{d}y}{\mathrm{d}t} + 12t^2y + 13t^3y = 61e^{\frac{1}{2}t}$	M1 A1
		4
7(b)	$m^2 + 4m + 13 = 0 \Rightarrow m = -2 \pm 3i$	M1
	$x = e^{-2t}\left(A\cos 3t + B\sin 3t\right)$	A1
	$x = ke^{\frac{1}{2}t} \Rightarrow \dot{x} = \frac{1}{2}ke^{\frac{1}{2}t} \Rightarrow \ddot{x} = \frac{1}{4}ke^{\frac{1}{2}t}$	B1
	$\frac{1}{4}k + 2k + 13k = 61 \Rightarrow k = 4$	M1 A1
	$t^3y = e^{-2t}\left(A\cos 3t + B\sin 3t\right) + 4e^{\frac{1}{2}t} \Rightarrow y = t^{-3}e^{-2t}\left(A\cos 3t + B\sin 3t\right) + 4t^{-3}e^{\frac{1}{2}t}$	M1 A1
		7

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Q6.

1	$m^2-8m-9=0 \Rightarrow m=-1,9$	M1
	$x = Ae^{-t} + Be^{9t}$	A1
	$x = ke^{8t} \Rightarrow \dot{x} = 8ke^{8t} \Rightarrow \ddot{x} = 64ke^{8t}$	B1
	$64ke^{8t} - 64ke^{8t} - 9ke^{8t} = 9e^{8t} \Rightarrow -9k = 9$	M1
	$k = -1$	A1
	$x = Ae^{-t} + Be^{9t} - e^{8t}$	A1
		6

Q7.

7(a)	$\frac{dy}{dx} + \frac{y}{\sqrt{(x^2+1)}} = \frac{x}{x^2+1}\left(x - \sqrt{(x^2+1)}\right)$	B1
	$e^{\int \frac{1}{\sqrt{(x^2+1)}} dx} = e^{\sinh^{-1} x}$	M1 A1
	$= x + \sqrt{(x^2+1)}$	A1
		4
7(b)	$\frac{d}{dx}\left(y\left(x+\sqrt{(x^2+1)}\right)\right) = -\frac{x}{x^2+1}$	M1 A1
	$y\left(x+\sqrt{(x^2+1)}\right) = -\int \frac{x}{x^2+1} dx = -\frac{1}{2}\ln\left(x^2+1\right) + C$	M1 A1
	$\ln 2 = C$	M1
	$y = \frac{\ln 2 - \frac{1}{2}\ln\left(x^2+1\right)}{x+\sqrt{(x^2+1)}} = \left(x - \sqrt{(x^2+1)}\right)\ln\left(\frac{1}{2}\sqrt{(x^2+1)}\right)$	M1 A1
		7

Q8.

2(a)	$9m^2 + 6m + 1 = 0 \Rightarrow m = -\frac{1}{3}$	M1	Auxiliary equation.
	$y = e^{-\frac{1}{3}x}(Ax+B)$	A1	Complimentary function.
	$y = p + qx + rx^2 \Rightarrow y' = q + 2rx \Rightarrow y'' = 2r$	B1	Particular integral and its derivatives.
	$18r + 6q + 12rx + p + qx + rx^2 = 3x^2 + 30x$	M1	Substitutes and equates coefficients.
	$r = 3, \quad q = -6, \quad p = -18$	A1	
	$y = e^{-\frac{1}{3}x}(Ax+B) + 3x^2 - 6x - 18$	A1	
		6	
2(b)	$y = 3x^2 - 6x - 18$	B1 FT	
		1	

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Q9.

4	$\frac{dy}{dx}+\frac{2}{x}y=\frac{1}{x}e^x$	B1	Divides through by x.
	$e^{2\int x^{-1}dx}=x^2$	M1 A1	Finds integrating factor, must be integrating x^{-1}.
	$\frac{d}{dx}\left(yx^2\right)=xe^x$	M1	Correct form on LHS and attempt to integrate $\frac{1}{x}e^x$ multiplied by *their* integrating factor.
	$yx^2=(x-1)e^x+C$	A1	
	$C=3$	M1	Finds C.
	$y=\frac{(x-1)e^x+3}{x^2}$	M1 A1	Divides through by coefficient of y.
		8	

Q10.

6	$m^2+8m+15=0\Rightarrow m=-3,-5$	M1	Auxiliary equation.
	$x=Ae^{-5t}+Be^{-3t}$	A1	Complementary function.
	$x=p\sin 3t+q\cos 3t\Rightarrow \dot{x}=3p\cos 3t-3q\sin 3t\Rightarrow \ddot{x}=-9p\sin 3t-9q\cos 3t$	M1 A1	Particular integral and its derivatives.
	$-9p-24q+15p=0\Rightarrow 6p-24q=0$ $-9q+24p+15q=102\Rightarrow 4p+q=17$	M1	Substitutes and equates coefficients.
	$p=4,\quad q=1$	A1	
	$x=Ae^{-5t}+Be^{-3t}+4\sin 3t+\cos 3t$	A1	
	$\dot{x}=-5Ae^{-5t}-3Be^{-3t}+12\cos 3t-3\sin 3t$	M1	Differentiating the correct form.
	$1=A+B+1$ $0=-5A-3B+12\Rightarrow A=6, B=-6$	M1 A1	Forms simultaneous equations using initial conditions.
	$x=6e^{-5t}-6e^{-3t}+4\sin 3t+\cos 3t$	A1	
		11	